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6000354600

Hughes's Matriculation Manuals.

KEY

TO

CATCH QUESTIONS

IN

ARITHMETIC & MENSURATION.

BY THE

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PREFACE.

In the following Key, the student is referred to the Hints at the end of the work itself whenever the question is practically solved there.

In some few instances, a different method of solution is given here from the one there, not necessarily as an improvement, but as an example of different ways of obtaining the same result.

Some of the multiplications and divisions are omitted to prevent the book becoming too bulky and expensive.

Much care has been bestowed to secure accuracy, but the author will be most grateful for the pointing out of any mistakes.

KEY

TO

CAPEL'S CATCH QUESTIONS IN ARITHMETIC.

CHAPTER I.

1. to 11. See Hints, pp. 305 and 306.

12.

347 (octonary)
8
---
28 eights
8
---
231 ones
Ans. 231 (denary).

13.

5)231 ones
---
5)46 fives 1 one
---
5)9 (five)^{2}s 1 five
---
1 (five)^3 4 (five)^{2}s
Ans. 1411 (quinary).

14.

41312 (quinary)
5
---
21 (five)^{3}s
5
---
108 (five)^{2}s
5
---
541 fives
5

2707 ones
Ans. 2707 (denary).

15.

9)2707 denary
---
9)300 nines 7 ones
---
9)33 (nine)^{2}s 3 nines
---
3 (nine)^{3}s 6 (nine)^{2}s
Ans. 3637 (nonary).

16. 36134 (septenary)
7
27 (seven)^{3}s
7
190 (seven)^{2}s
7
1333 sevens
7
9335 ones
Ans. 9335 (denary).

17. 4)9335 (denary)
4)2333 fours 3 ones
4)583 (four)^{2}s 1 four
4)145 (four)^{3}s 3 (four)^{2}s
4)36 (four)^{4}s 1 (four)^3
4)9 (four)^{5}s 0 (four)^{4}s
2 (four)^{6}s 1 (four)^5
Ans. 2101313 (quaternary).

18. 5718 (duodenary)
12
67 (twelve)^{2}s
12
805 twelves
12
9668 ones
Ans. 9668 (denary).

19. 216 (septenary)
7
15 sevens
7
111 ones
Ans. 111 (denary).

20. 2)111 (denary)
2)55 twos 1 one
2)27 (two)^{2}s 1 two
2)13 (two)^{3}s 1 (two)^2
2)6 (two)^{4}s 1 (two)^3
2)3 (two)^{5}s 0 (two)^4
1 (two)^6 1 (two)^5
Ans. 1101111 (binary).

21. 101101 (binary)
2
2 (two)^{4}s
2
5 (two)^{3}s
2
11 (two)^{2}s
2
22 (two)s
2
45 ones
Ans. 45 (denary).

22. 12)45 (denary)

3 twelves 9 ones

Ans. 39.

23 to 33. See Hints, p. 307.

34.

·36 ones
5

1·80 fifths
5

4·0 (fifth)²s

Ans. ·14 (quinary).

35.

·244 (quinary)
5)4· (fifth)³s

5)4·8 (fifth)²s

5)2·96 fifths

·592 ones

Ans. ·592 (denary).

36.

·75 (denary)
4

3·00 fourths

Ans. *·3 (quaternary).*

37.

·323 (quaternary)
4)3· (fourth)³s

4)2·75 (fourth)²s

4)3·6875 fourths

·921875 ones

Ans. ·921875 (denary).

38.

275·625 denary
8)275 ones

8)34 eights 3 ones

4 (eight)²s 2 eights

·625 ones
8

5·000 eighths

Ans. 423·5 (octonary).

39.

34·133 (quinary)
34 quinary
5

19 ones (denary)
5)3· (fifth)³s

5)3·6 (fifth)²s

5)1·72 fifths

·344 ones

Ans. 19·344 (denary).

40. 243·376 (denary)

5)243

5)48 fives 3 ones

5)9 (five)^{2}s 3 fives

1 (five)^{3}s 4 (five)^{2}s

·376 ones

5

1·880 fifths

5

4·400 (fifth)^{2}s

5

2·000 (fifth)^{3}s

Ans. 1433·142.

41. See Hints, p. 307.

42. 10 − 6 = 4 Ans.
10 − 9 = 1 „
10 − 4 = 6 „

43. 12 − 7 = 5 Ans.
12 − 4 = 8 „
12 − 8 = 4 „

44. 100 − 43 = 57 Ans.
100 − 2 = 98 „
100 − 77 = 23 „

45, 46. See Hints, p. 307.

47. Let 2347 be any number, and let 3274 be any number expressed by means of same digits.

2347 = some nines + 7,
and 3274 = „ + 7;
∴ the difference = some nines.

48. See Hints, p. 308.

49. If the 4 represent forty nines, the 0 will represent sevens and the 1 units; hence the unit point must be placed between the 1 and the 6.

50. See Hints, p. 308.

CHAPTER II.

1.

347612
39875

307737

Ans. 307737 found by subtraction.

2. See Hints, p. 308.

3.

100000

4761
Adding 398

Subtracting 5159

94841 Ans.

4.

47612
Adding 59163

106775 Ans.

5.

9734
Multiplying 379

87606
68138
Adding 57, 2920257

3689243 Ans.

6.

492)1000000(2032
984

1600
1476

1240
Subt. 492 256 984

236 Ans. 256

If 236 were added to dividend, the divisor would be contained exactly 2033 times.

7.

379)57432(151
379

1953
1895

582
379

203

Since 203 is more than the half of 379, we must add 379 − 203 or 176.

Ans. 57432 + 176 or 57608.

8.

791)10000(12
791

2090
1582

508

508 − 472 = 36

If then I take away 36 from dividend, I get what I want, viz.

10000 − 36 or 9664 Ans.

9. 5)347121 ones

7)69424 fives 1 one

9)9917 thirty-fives 5 fives

1101 (9 × 7 × 5)s 8 thirty-fives

Ans. 1101 and 8 × 35 + 5 × 5 + 1, or 1101 and remainder 306.

10 to 12. See Hints, p. 309.

13.

342615 octonary
423 „

1250247
705432
1613064

171633167 „
342615 „
12 denary

5241234 octonary.

14.

2 ones = 2
6 sevens $= 6\times(7-1)+6$
3 (seven)^{2}s $= 3\times(7^2-1)+3$
3 (seven)^{3}s $= 3\times(7^3-1)+3$
4 (seven)^{4}s $= 4\times(7^4-1)+4$,
but $(7-1)$, (7^2-1), etc., all contain 6 as a factor; hence 43362 (septenary)
= some sixes + 4 + 3 + 3 + 6 + 2
= some sixes + 18;
∴ the number will divide by 6.

15. 2 ones = 2
4 nines $= 4(9+1)-4$
3 (nine)^{2}s $= 3(9^2-1)+3$
5 (nine)^{3}s $= 5(9^3+1)-5$
4 (nine)^{4}s $= 4(9^4-1)+4$,
but $(9+1)$, (9^2-1), (9^3+1), etc., all contain 10 as a factor; hence 45342 (nonary) = some tens + 2 − 4 + 3 − 5 + 4 = some tens + 0; ∴ the number will divide by 10.

16. *See Hints, p. 310.*

17.

47169
172814412

566028 12 times
6792336 14400 „
81508032 172800000 „

8151482999628 Ans.

The 2nd line is the 1st × 1200, and the 3rd line is the 2nd × 12000.

18.

47169
152412

566028 12 times
3962196 8400 „
6792336 144000 „

7189121628 Ans.

The 2nd line is 1st × 700
„ 3rd „ 1st × 12000

19.

47169
1332221

518859 11 times
5707449 1210 „
62781939 1331000 „

62839532349 Ans.

The 2nd line is 1st line × 110
„ 3rd „ 2nd „ × 1100.

20.

Multiplying 399 × 95

1995
3591

37905

Subtracting 30

75)37875(505
375

375
375

505

Subtract 479

Ans. 26

21. (i.) 47 × 9 = 423

(ii.) 47 × (10 − 1)

470
Subt. 47

423

(iii.) 47 × 3 × 3 = 423

(iv.) 47 × 18

376
47

2)846

423

2.

£	s.	d.
50	0	0
45	6	7½
4	13	4½

23.

£	s.	d.
10	0	0
8	6	7½
1	13	4½

24, 25. See Hints, p. 310.

26. 15 miles an hour is a mile in 4 minutes; 10 miles an hour is a mile in 6 minutes; there is therefore a difference of 2 minutes in each mile, but the difference between being 1 hour too soon and one hour too late is 2 hours, and 2 hours ÷ 2 minutes = 60; therefore the distance is 60 miles. Ans.

27. B has £10 more than C, A has £17 + 10 more than C. Take away these £37 from £367, and divide remainder by 3.

£367 Ans. C's share £110
£ 37

3)330 „ B's „ £120

£110 „ A's „ £137.

28. B has walked 6 miles before A has started. They walk towards one another over the 44 miles left, at 4 + 3 or 7 miles an hour, hence they will meet in $\frac{44}{7}$ or $6\frac{2}{7}$ hours. Therefore they

meet $6\frac{2}{7} \times 4$ or $25\frac{1}{7}$ miles from London.

A walks the distance in $\frac{50}{4}$ or $12\frac{1}{2}$ hours. B will have walked in $12\frac{1}{2} + 2$ hours $14\frac{1}{2} \times 3$ miles or $43\frac{1}{2}$ miles, and will be therefore $6\frac{1}{2}$ miles from London when A arrives at Brighton.

29.

$$453\overline{)43251^{10}(533}$$

4033

2521

2243

2340

2243

53

Ans. $533\ \frac{53}{453}$.

Proof.

533
453

2443
4353
342053

432510

30. See Hints, p. 310.

31. 2)63 denary

2)31 twos 1 one

2)15 (two)^{2}s 1 two

2)7 (two)^{3}s 1 (two)^2

2)3 (two)^{4}s 1 (two)^3

1 (two)^5 1 (two)^4

63 (denary) = 111111 (binary)

12)107 denary

8 twelves 11 ones

107 (denary) = 8*e* (duodenary).

32. If 2 women have as much as 3 children, 1 woman has as much as $\frac{3}{2}$ children, and 1 man has as much as $\frac{35}{8}$ children.

24 men + 36 women + 41 children may be considered equal to

$$\frac{24 \times 35}{8} + \frac{36 \times 3}{2} + 41 \text{ children}$$

or 105 + 54 + 41 or 200 children

∴ each child receives $£\frac{100}{200}$ or $£\frac{1}{2}$

Each woman $\frac{3}{2}$ of $£\frac{1}{2}$ or 15s.

Each man $\frac{35}{8}$ of $£\frac{1}{2}$ or £2, 3s. 9d.

33. Each man has 7 + 5 or 12s. more than a child, therefore the 20 men have £12 beyond an equal division. Each woman has 5s. more than a child, therefore 15

women have £3, 15s. beyond an equal division. If then we subtract £15, 15s. from £99, 9s. and divide the remainder by 20 + 15 + 27, we have the share of each child.

£	s.		£	s.	£
99	9	62	83	14	1
Subt. 15	15		62		
£83	14		21		
			20		
			434		7s.
			434		

Each child has £1 7s.
„ woman „ £1 12s.
„ man „ £1 19s.

34. If 15 women receive £5, 5s., each woman receives 7s. Each child receives 7 − 5 or 2s., and each man 7 + 7 or 14s. Therefore 20 men, 15 women, and 27 children receive

$$£\frac{20 \times 14}{20} + £5\tfrac{1}{4} + £\frac{27 \times 2}{20}$$

or £14 + £5 5s. + £2 14s.
or £21 19s. Ans.

35. A is on his machine $6 - \frac{1}{2}$ or $5\frac{1}{2}$ hours. He therefore travels at $55 \div 5\frac{1}{2}$ or 10 miles an hour. B goes in 2 hours what A does in 3 hours, or B goes 15 miles an hour, but he only travels 5 miles more at this pace; he therefore has done 35 miles and has 20 more *to go, which he* does in 2 hours, and therefore arrives at Cambridge at 3 P.M.

Note.—For 'and' in 3rd line of Question read 'who.'

36. If B had neither altered his pace nor dined, he could have arrived at Cambridge $\frac{5}{15}\frac{5}{}$ hours after 10, *i.e.* at 1 hr. 40′. For A to have arrived at Cambridge at 3 hrs. 40′, since from 9 hrs. to 3 hrs. 40′ is 6 hrs. 40′, and, at 10 miles an hour, A would be $\frac{5}{10}\frac{5}{}$ or $5\frac{1}{2}$ hours on his machine; he therefore would have to wait for dinner 6 hrs. 40′ less 5 hrs. 30′ or 1 hr. 10′.

37. A has £10 more than B
C „ £5 „ B
D „ £8 less than £5 more than B, or £3 less than B; we must then take away from £108 £10 + 5 − 3 or £12.

Therefore B's share is $\frac{£108 - 12}{4}$ or £24.

38. A has £34, B has £24, C has £29, and D has £21, or altogether there are £108.

39. A has £34, B has £24, therefore C and D have 108 − 58 or £50. Let us take C's extra £8 and divide the £42 left by 2, and we get C's share as $\frac{42}{2} + 8$ or £29. Therefore B has £29 − 24 or £5 less than C.

40. A and E have 2 shares,
B „ D „ 2 „
C has 1 share; therefore C has $£\frac{100}{5}$ or £20, and B and D have £40; therefore D has £40 − 15 or £25.

41. See Hints, p. 311.

42. A, D, and E have £100 − £35 or £65, B and D have half of A, B, D, E, or $\frac{1}{2}$ £(65 + 15) or £40; ∴ D has £25 and £35 − £25 = £25 − £15.

43. The student is recommended to draw the lines, measuring them carefully by some scale.

44, 45. See Hints, p. 311.

46.
374*t* (undenary)
23 „
10038
7399
Ans. 83*t*18 (undenary).

47.
4)47622 (octonary)
7)11744 fours 2 ones
1327 3 fours
Ans. 1327 (octonary) with remainder 16 (octonary).
Note. 3 fours + 2 = 14 *(denary)* = 16 *(octonary)*.

48.
4)37270 (nonary)
7)8516 fours 3 ones
1202 1 four.
Ans. 1202 (nonary) and 7 remainder.

49.
11)3751 (denary)
11)341
31
Factors are 11 . 11 . 31 (denary), or 14 . 14 . 43 . (septenary).
14 (septenary)
14 „
62
14
232 (septenary)
43 „
1026
1261
13636 (septenary)
13636 (septenary)
7
10 (seven)^{2}s
7
76 (seven)^{2}s
7
535 sevens
7
3751 (denary).

50.

343)362141(1034

343		343 times
1614	1 =	343 „
1362	2 =	1016 „
2221	3 =	1362 „
2035	4 =	2035 „
153	5 =	2411 „
	6 =	3054 „

Ans. $1034\frac{153}{343}$ (septenary).

CHAPTER III.

1. qrs. lbs. Av.

3 17

28

101 lbs.

7000

24 { 4)707000 grs.

6)176750 four grs.

2\0)2945\8 dwts. 2 four grs.

12)1472 oz. 18 dwts.

122 lbs. 8 oz.

Ans. 122 lbs. 8 oz. 18 dwts. 8 grs.

2. cwt. qrs. lbs. oz.

1 1 17 8

4

5 qrs.

28

$157\frac{1}{2}$ lbs.

7000

1099000

3500

24 { 4)1102500 grs.

6)275625 four grs.

2\0)4593\7 dts. 3 four grs.

12)2296 oz. 17 dwts.

191 lbs. 4 oz.

Ans. 191 lbs. 4 oz. 17 dts. 12 grs.

3. lbs. oz. dwts.
3 7 15
12
—
43 oz.
20
—
875 dwts.
24
—
3500
1750
—
7\000)21\000 grs.
—
3 lbs.

Ans. 3 lbs. Av.

4. See Hints, p. 312.

5. drs. scr. grs.
6 2 8
3
—
20 scr.
20
—
24 { 4)408 grs.
6)102 four grs.
—
17 dwts.

Ans. 17 dwts.

6. 2s. 10d. = 34d.,
2s. $1\frac{1}{2}$d. = $25\frac{1}{2}$d.

$$\frac{108 \times 34}{25\frac{1}{2}}, \text{ or } \frac{\overset{36}{\cancel{108}} \times \overset{2}{\cancel{34}} \times 2}{\cancel{51}\ \cancel{3}}$$

or 144 coins worth 2s. $1\frac{1}{2}$d. each.

7. £409, 10s. = 8190s.
2s. 3d. = $2\frac{1}{4}$s. and
1s. 1d. = $1\frac{1}{12}$s.

$$\frac{8190}{2\frac{1}{4}} = \frac{8190 \times 4}{9} = 3640.$$

Ans. 3640 lbs. of tea.

$$\text{And } \frac{8190}{1\frac{1}{12}} = \frac{8190 \times 12}{13} = 7560.$$

Ans. 7560 lbs. of coffee.

1 lb. of tea and 1 lb. of coffee cost $2\frac{1}{4} + 1\frac{1}{12}$ shillings. Then for equal quantities

$$\frac{8190}{2\frac{1}{4} + 1\frac{1}{12}}, \text{ or } \frac{8190 \times 12}{27 + 13},$$

or 2457 lbs. of both tea and coffee.

8.

po.	yd.	ft.
1	1	$1\frac{3}{4}$
		$30\frac{1}{4}$
		$31\frac{1}{4}$ yds.
		9
		283 ft.

ac.	ro.	po.	yds.	ft.
5	2	17	2	$2\frac{3}{4}$
4				
22 ro.				
40				
897 po.				
$30\frac{1}{4}$				
26912				
$224\frac{1}{4}$				
$27136\frac{1}{4}$ yds.				
9				
244229 ft.				

$283)244229(863$
2264

1782
1698

849
849

Ans. 863 plots.

9. A walks $(4 \times 365 + 1) \times 3 \times 3\frac{1}{2}$ miles, and B rides $(4 \times 365 + 1) \times 2 \times 9\frac{3}{4}$ miles, and $2 \times 9\frac{3}{4} - 3 \times 3\frac{1}{2} = 19\frac{1}{2} - 10\frac{1}{2}$, or 9.

B, therefore, will have gone over $9 \times (4 \times 365 + 1)$ more than A, or 13149 *miles*.

10.

1 far. × 935 = 4)935 fars.
233d. ¾
2d. × 935 = 1870d.
12)2103d.
175s. 3d.
17s. × 935 = 15895s.
20)160710s.
£803, 10s.
£43 × 935 = £40205
£41008
Ans. £41008, 10s. 3¾d.

Proof—

£	s.	d.	£
935)41008	10	3¾	(43
3740			
3608			
2805			
803			
20			
16070			(17s.
935			
6720			
6545			
175			
12			
2103			(2d.
1870			
233			
4			
935			(1 far. or q.
935			

11.

9)51 ft. or 1 ft. × 51
5 yds. 6 ft.
17 yds. × 51 = 867 yds.
30¼)872
28 po. 25 yds.
11 po. × 51 = 561 po.
40)589 po.
14 ro. 29 po.
2 ro. × 51 = 102 ro.
4)116 ro.
29 ac. 0 ro.
3 ac. × 51 = 153 ac.
182 ac.

Ans. 182 a. 0 r. 29 p. 25 yds. 6 ft.

The proof by division can easily be tested by comparing many of the numbers with those found in the multiplication, as is evident in Ex. 10.

I divided by 30¼ in one effort, thus :—

30¼)872(2
605
267

Since 60½ tens = 605,
and 30¼)267(8
242
25

12. B rides in the given time $(4 \times 365 + 1) \times 9\frac{3}{4} \times 2$, or $28489\frac{1}{2}$ miles. A therefore walks $28489\frac{1}{2} - 13149$, or $15340\frac{1}{2}$ miles, and this $\div$ by 1461×3 will give $3\frac{1}{2}$ miles as his daily average. Ans.

13. See Hints, p. 312.

14. $504 \times 365 + \frac{504}{4}$ divided by $7 = 72 \times 365 + 18 = 26298$.

15. From Feb. 29, 1880, to Oct. 19, $1884 = 365 \times 4 + 1 + 233$, or 1694 days, or 7×242 days; hence Feb. 29, 1880, was on a Sunday. Between Feb. 29, 1880, and Sept. 17, 1886, there are $365 \times 6 + 1 + 201$, or 2392 days, and these divided by 7 gives us 341 weeks and 5 days, of which the first is a Sunday. Therefore Ans. 342 Sundays.

To find the Sundays in March 1885 :—

From Oct. 19 (a Sunday), 1882, to Feb. 28, 1885, there were 132, or $7 \times 18 + 6$ days. Feb. 28 was therefore a Saturday, and the Sundays in March 1885 were on the 1st, 8th, etc.

16. See *Hints, p. 312.*

17.

$$\begin{array}{rr} & 15^\circ \\ \text{subtract} & 0^\circ \\ \hline & 15^\circ \end{array}$$

$15 \times 4 = 60^\circ$ or 1 hr.

Therefore it is 11 o'clock a.m. at 15° w., when noon at 0°.

18. To find the difference between 23° 30′ E. and 23° 30′ W. we must add; result, 47°.

$47 \times 4 = 188$, or 3 hr. 8′.

$$\begin{array}{rr} & 4 \text{ hr. } 25' \\ \text{Subtract} & 3 \text{ hr. } 8' \\ \hline & 1 \text{ hr. } 17' \end{array}$$

Ans. 1 hr. 17′ p.m.

19. If ship is sailing to w., it travels with the sun, therefore its days are longer, and the clock must be put back 4×11 or 44 min. each day. Conversely, travelling the other way, the clock must be put forward 44 min. each day.

20. $35' \ 30'' = 35\frac{1}{2}$, and this divided by 4 gives $8\frac{7}{8}$ degrees.

Ans. 8° 52′ 30″.

21. 1 fur. 5 po. 2 yds.
40
45 po.
$5\frac{1}{2}$
$249\frac{1}{2}$ yds.
3
$748\frac{1}{2}$ ft.
12
·09 in.)8982·00 in.
99800
Ans. 99800 times.

22. ·03 oz.)7·0721 oz.
235 . . . 221

Ans. 235 times, with a remainder of 221 tenths of thousandths of an oz.

23. 3)357 thirds or $\frac{1}{3}$
119 grs. × 357
3 grs. × 357 = 1071 grs.
24)1190 grs.
49 dwts. 14 grs.
5 dts. × 357 = 1785 dwts.
20)1834
12)91 oz. 14 dwts.
7 lbs. 7 oz.
Ans. 7 lbs. 7 oz. 14 dwts. 14 grs.

24.

		£	s.	d.	=	£	s.	d.
77 saddles	at	0	18	$8\frac{1}{2}$	=	72	0	$6\frac{1}{2}$
317 sets of harness	at	3	17	6	=	1228	7	6
15 carriages	at	19	12	1	=	294	1	3
7 coaches	at	117	4	$8\frac{1}{2}$	=	820	12	$11\frac{1}{2}$
						2415	2	3

Ans. £2415, 2s. 3d.

I may mention that in finding the third line I multiplied £$19\frac{5}{8}$ by 15, and subtracted 5 × 15d. (12s. 1d. being 12s. 6d. − 5d.).

25. *See Hints, p. 313.*

26. From 4 p.m. Jan. 1, to noon June 10, is 4 hrs. less than 160 days, or $159\frac{5}{6}$ days and 2 hrs. 25 min., which the clock has gained, divided by this no. $\frac{959}{6}$ will give us the answer.

2 hrs. 25 min.
60
145 min.
60
8700 sec.
6
959)52200 sixths of a sec.
4795(54 sec.
4250
3836
414

Ans. $54\frac{414}{959}$ sec. a day.

27. From Feb. 29 to April 3 is 34 days, in which the clock lost 51 min.; from April 3 to June 3 is 61 days, in which the clock gained $30\frac{1}{2}$ min.; therefore from Feb. 29 to June 3 it lost $51 - 30\frac{1}{2}$, or $20\frac{1}{2}$ min. Hence it showed, at noon on June 3, 11 hrs. 39′ 30″.

28. Altogether the clock has lost $20\frac{1}{2}$ min. Had the regulator not been touched at all, the *clock would have lost* $95 \times 1\frac{1}{2}$ min., or $142\frac{1}{2}$ min. Of these $142\frac{1}{2}$ min., $142\frac{1}{2} - 20\frac{1}{2}$, or 122, are made up by the change in rate caused by the touching of the regulator. The difference between losing $1\frac{1}{2}$ min. and gaining $\frac{1}{2}$ min. is 2 min.; therefore the regulator must have been altered $122 \div 2$, or 61 days, and 61 days before June 3 is April 3, which we had to find.

29. From April 3 to June 3 the clock has gained $30\frac{1}{2}$ minutes; but it has lost $20\frac{1}{2}$ minutes at the end. Altogether, therefore, it has lost $30\frac{1}{2} + 20\frac{1}{2}$, or 51 min. 51 min. contains $1\frac{1}{2}$ min. 34 times; therefore the clock began to lose 34 days before April 3, or last day of February.

30. Between April 3 and June 3 the clock has gained $\frac{1}{2} \times 61$, or $30\frac{1}{2}$ min.; but it has altogether lost $30\frac{1}{2} + 20\frac{1}{2}$, or 51 min., and this $\div$ 34 (number of days between February 29 and April 3) gives us $1\frac{1}{2}$ min. as the Answer.

31. From February 29 to April 3 the clock had lost $34 \times 1\frac{1}{2}$, or 51 min., and of these 51 min. it made up (by gaining $\frac{1}{2}$ a min. each day) $51 - 20\frac{1}{2}$ (the minutes eventually lost), or $30\frac{1}{2}$ min.; and

since $\frac{1}{2}$ is contained in $30\frac{1}{2}$ sixty-one times, the day on which it pointed to 11 h. 39′30″ at noon was 61 days after April 3, *i.e.* June 3. Ans.

32. From February 29 to April 3 the clock has lost $34 \times 1\frac{1}{2}$, or 51 min.; but altogether it has only lost $20\frac{1}{2}$ min. Therefore in the 61 days between April 3 and June 3 it must have gained $51 - 20\frac{1}{2}$, or $30\frac{1}{2}$ min., which gives us $\frac{1}{2}$ a min. a day as our Answer.

33. See Hints, p. 313.

34. He pays 1d. × 14 = 1s. 2d.

1 stone
14
—
14 lbs.
7000
—
2\0)9800\0 grs.
—
3)4900 scr.
—
8)1633$\frac{1}{3}$ drs.
—
$204\frac{1}{8}$ oz.
12)$204\frac{1}{8}$ pence
—
17s. o$\frac{1}{8}$d.

Profit, 17s. o$\frac{1}{8}$d. − 1s. 2d., or 15s. 10$\frac{1}{8}$d. Ans.

35. He gains 125 × 10 pence, or 1250d., and loses 200 × 3 pence, or 600d.; therefore he gains 650d.

12)650d.
—
2\0)5\4s. 2d.
—
£2, 14s.
Ans. £2, 14s. 2d.

This is a better solution than the one given in the Hints.

36. From October 24, 1836, to October 24, 1884, is 365 × 48 + 12 days; and from February 29 to October 24 is 238 days: we want to find what the remainder is after dividing all these days by 7. Let us write 365, 364 + 1. Then the days are 364 × 48 + 48 + 12 + 238, which contains some sevens + 4 (364 being 7 × 52, and 238 being 7 × 34, and 48 + 12 being 7 × 8 and 4). Now, 4 days before a Friday is a Monday. Ans.

37. See Hints, p. 314.

38. Since the journey takes 1 hour, and 60 minutes contains 10 minutes 6 times, there are 6 omnibuses always on the route going one way and 6 going the other.

Ans. 12.

39. See Hints, p. 314.

40. His cash payments, viz. £300, are $(1-\frac{1}{6})$ of $(1+\frac{1}{5})$, or $\frac{10}{9}$ of buying price; therefore what he sold for £300 he paid $\frac{9}{10}$ of 300, or £270. His profits then were £30. But all his profits were £225$\frac{1}{5}$. Therefore his profits on his booked sales were £195$\frac{1}{5}$. £1's worth he marks £1, 6s. 8d., and of this he receives $(1-\frac{1}{20})$. Now, £$(\frac{19}{20}$ of $\frac{4}{5})$ = £$\frac{19}{15}$. His profit, then, on each £1 is £$\frac{4}{15}$, and £$\frac{4}{15}$ is contained in £195$\frac{1}{5}$ 732 times; hence his outlay on this part of his transaction was £732, which, with £270, gives us £1002. Ans.

41. He receives altogether £1002 and £225$\frac{1}{5}$, or £1227$\frac{1}{5}$, of which he receives £300 for his cash sales, and this leaves £927$\frac{1}{5}$ or £$\frac{4636}{5}$ for his booked sales; but this £$\frac{4636}{5}$ is only $\frac{19}{20}$ of the marked price, and the £300 is $\frac{5}{6}$ of the marked price. Therefore the marked price of all is £($\frac{6}{5}$ of 300 + $\frac{20}{19}$ of $\frac{4636}{5}$), or £360 + £976, or £1336, and £1336 − £1002 is £334, or $\frac{1}{3}$ of £1002. He therefore marks them at a profit of $\frac{1}{3}$ of cost price, or 4d. in the shilling.

42. As in 41, he receives altogether £1227$\frac{1}{5}$, of which £300 is for cash sales. The rest, viz. £927$\frac{1}{5}$, is $\frac{19}{20}$ of $\frac{4}{3}$ of cost price. Therefore the cost price of booked sales is £($\frac{15}{19}$ of $\frac{4636}{5}$), or £732. Therefore what he sold for £300 he gave £1002 − £32, or £270. These were marked $\frac{4}{3}$ of £270, or £360. He therefore allowed £60 discount on £360, or one-sixth —that is, 2d. in the shilling.

43. This resolves itself into the following problem:—

Divide £1002 into two parts, so that $\frac{1}{9}$ (*i.e.* $\frac{5}{6}$ of $\frac{4}{3}$ − 1) of the one and $\frac{4}{15}$ (*i.e.* $\frac{19}{20}$ of $\frac{4}{3}$ − 1) of the other is = £225$\frac{1}{5}$. $\frac{4}{15}$ of £1002 is £$\frac{13386}{5}$, or 267$\frac{1}{5}$, which is £42 more than £225$\frac{1}{5}$. If, then, I divide £42 by $\frac{4}{15}-\frac{1}{9}$, or $\frac{7}{45}$, I find the number which must be multiplied by $\frac{1}{9}$ to produce the required result.

$$£42 \div \tfrac{7}{45} = £270.$$

And on this he made $\frac{1}{9}$ of itself; therefore the money he received was 1$\frac{1}{9}$ of £270, or £300. Problems of this nature are fully explained in Chap. XVII.

44.

	£1002	purchase money
	$225\frac{1}{5}$	profits
	$1227\frac{1}{5}$	receipts
Subtracting	300	ready money payments
	$927\frac{1}{5}$	receipts of booked sales.

Again,

	£1002	purchase money
$\frac{9}{10}$ of £300	270	cost of cash sales
	£732	cost of booked sales.

$\frac{4}{3}$ of £732, or £976 = booked sales. His losses, therefore, are £976 − $927\frac{1}{5}$, or £$48\frac{4}{5}$, and this is contained $\frac{1}{20}$ of a time in £976, or his bad debts are 1s. in the £.

45. A pays altogether £$3 \times 52 \times \frac{1}{4} + £12 \times 52 \times \frac{1}{2}$, or £351, and earns with it £$12 \times 52 \times \frac{5}{4}$, or £780, making a profit of £429. B pays altogether £$12 \times 52 \times \frac{3}{10}$, or £187, 4s., and only earns with it £$10 \times 52 \times 1$, or £520, making a profit of £332, 16s. Therefore A's profits are £96, 4s. more than B's.

46. See Hints, p. 314.

47. A's profits can be found, as in **45**, to be £429. Therefore B's are £429 − £96, 4s., or £332, 16s.; but it cost him £$12 \times 52 \times \frac{3}{10}$, or £187, 4s. to keep the horse. He therefore earned in 10 years £332, 16s. + £187, 4s., or £520, and this proves that it worked 10 years, or was 2 years old *when it began to work.*

48. A's profits can be found, as before, to be £429; and hence B's to be £332, 16s. Of this it has cost him £$2 \times 52 \times \frac{3}{10}$, or £$31\frac{1}{5}$, before it earns anything. After it works, it makes as profit for B £$1 - \frac{3}{10}$, or £$\frac{7}{10}$, a week, and altogether it makes for him £$31\frac{1}{5} + £332\frac{4}{5}$, or £364, and £$\frac{7}{10}$ is contained in this 520 times; hence it worked 10 years, or was 12 years old when it died.

49. B's profit can be found, as before, to be £332, 16s., and his actual receipts are £10×52, or £520; hence the horse's keep was £520 − £332, 16s., or £187, 4s., and this divided by 52×12 gives us 6s. a week.

50. B's profits can be found as before, and hence A's are £429; but it cost A $3 \times 52 \times \frac{1}{4}$ + £$12 \times 52 \times \frac{1}{2}$, or £351, to keep the horse. Hence his earnings were £429 + 351, or £780, and this divided by 12×52 gives £$\frac{5}{4}$, or 25s. a week.

CHAPTER IV.

1 to 4. See Hints, p. 315.

5. (i.) $3245\frac{3}{4}$ art. at

			£	s.	d.
£1	=		3245	15	0
10s.	$\frac{1}{2}$	£1	1622	17	6
5s.	$\frac{1}{2}$	10s.	811	8	9
3s. 4d.	$\frac{1}{3}$	10s.	540	19	2
6d.	$\frac{1}{10}$	5s.	81	2	$10\frac{1}{2}$
3d.	$\frac{1}{2}$	6d.	40	11	$5\frac{1}{4}$
Ans.			£6342	14	$8\frac{3}{4}$

(ii.) $3245\frac{3}{4}$ art. at

			£	s.	d.
£1	=		3245	15	0
£2			6491	10	0
10d.	$\frac{1}{24}$	£1	135	4	$9\frac{1}{2}$
1d.	$\frac{1}{10}$	10d.	13	10	$5\frac{3}{4}$
Less 11d.			148	15	$3\frac{1}{4}$
Ans.			£6342	14	$8\frac{3}{4}$

6. See Hints, p. 315.

7. 375 tons at

			£	s.	d.
£1	=		375	0	0
£6			2250	0	0
Less 2s. 6d.	$\frac{1}{8}$	£1	46	17	6
Ans.			£2203	2	6

8. $1004\frac{2}{3}$ lbs. at

			£	s.	d.
£1	=		1004	13	4
£3			3014	0	0
2s.	$\frac{1}{10}$	£1	100	9	4
1s.	$\frac{1}{2}$	2s.	50	4	8
Less 3s.			150	14	0
Ans.			£2863	6	0

9. 4 tons 19 cwt. 3 qrs. 21 lbs. = 5 tons less 7 lbs.

			£	s.	d.
1 ton	costs		5	6	8
5 tons			26	13	4
1 cwt.	$\frac{1}{20}$	1 ton	0	5	4
7 lbs.	$\frac{1}{16}$	1 cwt.	0	0	4

Ans. £26 13 0

10. See Hints, p. 315.

11.

£	s.	d.
3	4	$7\frac{1}{3}$
		7
£22	12	$3\frac{1}{3}$ Ans.

$\frac{1}{3} \times 7 = \frac{7}{3} = 2\frac{1}{3}$, of which the 2 is carried to the 7 × 7d.

12.

£	s.	d.
7)3	4	$7\frac{1}{3}$
3)0	9	$2\frac{1}{21}^{6}$
5)0	3	$0\frac{5}{63}^{8}$
0	0	$7\frac{1}{315}^{21}$ Ans.

13.

fur.	po.	yds.	ft.
0	5	2	$1\frac{1}{3}$
			4
21		$3\frac{1}{2}$	$2\frac{1}{3}$
			7
3	32	2	$1\frac{1}{3}$ Ans.

14.

sq.	po.	yds.	ft.	in.
5)5	17	2	0	
7)1	3	4	0	
Ans. 4		6	108	

15. $342\frac{1}{3}$ (septenary)

5

$2404\frac{2}{3}$ Ans.

16. 11)$4213\frac{2}{3}$ (senary) ($343\frac{2}{33}$

33

51

44

33

33

$\frac{2}{3}$

Ans. $343\frac{2}{33}$

17. 3)$53217\frac{2}{3}$ (nonary)

7)$17065\frac{5}{10}$

2)$2260\frac{55}{70}$

$1130\frac{27}{70}$ Ans.

18. t)$4t5e3\frac{3}{t}$ (duodenary)

3)$5t23\frac{79}{84}$

7)$1e49\frac{27}{84}$

$341\frac{29}{84}$

19.

	ac.	ro.	po.	yd.	ft.	in.
	5)4	0	2	0	0	2
Ans.	3	8	12	0	130	

20.

	cost		s.	d.
1 ro.	cost		1	0
16 ro.	,,		16	0
2 po.	$\frac{1}{20}$	1 ro.	0	$0\frac{3}{5}$
1 yd.	$\frac{2}{121}$	2 po.	0	$0\frac{6}{605}$
1 ft.	$\frac{1}{9}$	1 yd.	0	$0\frac{2}{1815}$
1 in.	$\frac{1}{144}$	1 ft.	0	$0\frac{1}{180880}$
		Ans.	16	$0\frac{78409}{130680}$

21.

6)3315 (septenary)

6)402

3)45

14

Ans. 2.3.2.3.3.14.

14 (septenary) or 11 (denary) being a prime.

Note. — Since $3+3+1+5 = 12$ or 6×2, I saw at once that the number was divisible by 6 (one less than the radix of scale).

22. See Hints, p. 315.

23.

8)4741 (nonary)

8)538

5)61

12

Ans. 2.2.2.2.2.2.5.12.

12 (nonary) or 11 (denary) being a prime number.

24. See Hints, p. 316.

25.

12)5214 ones (octonary)

12)341 twelves 0 ones

12)22 (twelve)^{2}s 9 twelves

1 (twelve)^3 6 (twelve)^{2}s,

and $\frac{1}{3} = \frac{4}{12} = \cdot 4$;

$\therefore$ *Ans.* 1690·4 (duodenary).

26. 9)657 ones (octonary)

9)57 nines 8 ones

5 (nine)^{2}s 2 nines,

or 675 (octonary) = 528 (nonary)

$528\overline{)78121}(143$

528		528 times
	1 =	528 „
2522	2 =	1157 „
2325	3 =	1686 „
	4 =	2325 „
1861	5 =	2854 „
1686	6 =	3483 „
	7 =	4122 „
164	8 =	4651 „

Ans. $143\frac{164}{528}$ (nonary).

27.

			lbs.	oz.	dwts.	grs.
1256 sov. at 1 oz.	=		104	8	0	0
5 dwts.	$\frac{1}{4}$ oz.		26	2	0	0
3 grs.	$\frac{1}{10}$	5 dwts.	0	7	17	0
$\frac{1}{2}$ gr.	$\frac{1}{8}$	3 grs.	0	1	6	4
		Ans.	26	11	3	4

28.

dwts.	grs.	lbs.
5	$3\frac{1}{2}$	185
24		7000
123 grs.		1295000 grs.
2		2
247 half grs.		2590000 half grs.

$$
\begin{array}{r|l}
247 & 2590000 \\
247 & \\
\hline
 & 1200 \\
 & 988 \\
\hline
 & 2120 \\
 & 1976 \\
\hline
 & 1440 \\
 & 1235 \\
\hline
 & 205
\end{array}
\quad 10485
$$

Ans. $10485\frac{205}{247}$.

The answer in second edition is wrong.

29. $\frac{1}{5}-\frac{1}{6}=\frac{6-5}{30}=\frac{1}{30}$ Ans.

30. One division is $\frac{1}{9}$ of an inch, the other is $\frac{1}{11}$ of an inch; therefore the distance between them is $\frac{1}{9}-\frac{1}{11}$ or $\frac{2}{99}$ in. Ans.

Similarly the distance between the 2nd division of the one and the 2nd of the other will be $\frac{2}{9}-\frac{2}{11}$ or $\frac{4}{99}$. Ans.

31. $£5\frac{1}{3}\div£\frac{1}{6}=32$ times; $5\frac{1}{3}\times 20\text{s.}\div 5\frac{1}{3}\text{s.}=20$ times, $32-20=12$ times. Ans.

32. The remainder is 315×417 or 131355, and $1000000-131355=868645$. Ans.

33 to 47. See Hints, pp. 316, 317, and 318.

48. We can keep the numbers much smaller by calculating the expenses of an acre.

Rent and taxes $\frac{5}{4}$ of 35s. or $43\frac{3}{4}$s.

Wages $\frac{5\times 15\times 52}{600}$ or $6\frac{1}{2}$

Harvesting	$81\frac{3}{4}$
Profit	20
	152s.

and to receive this with grain at 32s. a quarter, I must have $\frac{152}{32}$, or $4\frac{3}{4}$ quarters to each acre. Ans.

49. $£148-\frac{235}{240}$ of 151, or $£148-£147$, 17s. 1d., or better by 2s. 11d. Ans.

CHAPTER V.

1 to 10. See Hints, pp. 318, 319.

11.

6439	8131
5076	6439
1363	1692
1316	1363
47	329
	329

Ans. 47.

12. $7429 = 17 \cdot 19 \cdot 23$.
The two numbers whose L.C.M. is 7429 and G.C.M. 17, are 17×19, and 17×23.

Ans. 17×23, or 391.

13. (i.) 25)2475
9)99
11

(ii.) 8)37224
11)4653
9)423
47

(iii.) 2)962
13)481
37

Ans. (i.) $5 \cdot 5 \cdot 3 \cdot 3 \cdot 11$.
„ (ii.) $2 \cdot 2 \cdot 2 \cdot 11 \cdot 3 \cdot 3 \cdot 47$.
„ (iii.) $2 \cdot 13 \cdot 37$.

14.
$$\frac{17}{2703} - \frac{1}{2567} = \frac{2567 - 159}{17 \cdot 159 \cdot 151}$$
$$= \frac{2408}{17 \cdot 159 \cdot 151}, \text{ or } \frac{2408}{408153}.$$
Ans.

15.
$$5 - \frac{1}{3} - \frac{3}{5} - \frac{5}{7} - \frac{7}{9} - \frac{9}{11}$$
$$= \frac{17325 - 1155 - 2079 - 2475 - 2695 - 2835}{3 \cdot 5 \cdot 7 \cdot 3 \cdot 11}$$
$$= \frac{6086}{3465} \text{ or } 1\frac{2621}{3465}. \text{ Ans.}$$

16. $\frac{4}{5}$ of $3\frac{1}{3}+2\frac{1}{5}\times\frac{4}{33}-\frac{1}{2}\div\frac{5}{4}$

$+2\frac{3}{3\frac{1}{3}}=\frac{4}{5}$ of $\frac{10}{3}+\frac{11}{5}\times\frac{4}{33}-\frac{1}{2}$

$\times\frac{4}{5}+2\frac{9}{10}=2\frac{2}{3}+\frac{4}{15}-\frac{2}{5}+2\frac{9}{10}$

$=4\frac{20+8+27-12}{3\cdot5\cdot2}=4\frac{43}{30}$

$=5\frac{13}{30}$. Ans.

17. (i.)

$\frac{£3,\ 4s.\ 7d.}{£5,\ 9s.\ 4\frac{1}{2}d.}=\frac{64s.\ 7d.}{109s.\ 4\frac{1}{2}d.}$

$=\frac{775d.}{1312\frac{1}{2}d.}=\frac{775\times2}{2625}$ or $\frac{62}{105}$. Ans.

(ii.) $\frac{£3,\ 16s.\ 6d.}{£8,\ os.\ 8d.}=\frac{76s.\ 6d.}{160s.\ 8d.}$

$=\frac{76\frac{1}{2}s.}{160\frac{2}{3}s.}$ or $\frac{459}{964}$.

(iii.) $\frac{7\text{ lbs. }2\text{ oz. }1\text{ dwt. }4\text{ grs.}}{9\text{ lbs. }7\text{ oz. }0\text{ dwt. }12\text{ grs.}}$

$=\frac{86\text{ oz. }1\text{ dwt. }4\text{ grs.}}{115\text{ oz. }0\text{ dwt. }12\text{ grs.}}$

$=\frac{1721\frac{1}{6}\text{ dwts.}}{2300\frac{1}{2}\text{ dwts.}}=\frac{10327}{13803}$. Ans.

18. To find the L.C.M. of $2\frac{1}{4}$s., $3\frac{1}{3}$s., $4\frac{1}{6}$s., $6\frac{3}{4}$s., or 9, 10, 25, 27, it is $3\cdot3\cdot2\cdot5\cdot5\cdot3\cdot$s., or £67, 10s. If worked in pence the L.C.M. is $3\cdot3\cdot2\cdot5\cdot5\cdot3\cdot3\cdot2\cdot2\cdot$d., or same as before.

19. $3\frac{4}{5\frac{1}{8}}\div2\frac{3}{4\frac{1}{8}}=3\frac{24}{31}\div2\frac{5}{7}$

$=\frac{117}{31}\times\frac{7}{19}=\frac{819}{589}=1\frac{230}{589}$.

20. G.C.M. of $131\frac{2}{3}$ and $417\frac{2}{9}$ is the G.C.M. of 395 and 3755 $\div$ 9.

$$\frac{395\ 3755}{79\ 751}$$

∴ G.C.M. is $\frac{5}{9}$, which is contained in $131\frac{2}{3}$, 711 times, and in $417\frac{2}{9}$, 6759 times. The L.C.M. is $395\div5\times3755$, or 296645.

21, 22. See Hints, p. 320.

23. $\frac{63\text{ gals. }3\text{ qts.}}{\frac{3}{4}\text{ pt.}}=\frac{255\times2\text{ pts.}}{\frac{3}{4}\text{ pt.}}$

$=\frac{255\times2\times4}{3}=680$. Ans.

24. $\frac{3\text{ ro. }5\text{ po. }7\text{ yds.}}{2\frac{1}{4}\text{ ft.}}$

$=\frac{3788\frac{1}{4}\text{ yds.}}{2\frac{1}{4}\text{ ft.}}=\frac{3788\frac{1}{4}\times9}{2\frac{1}{4}}$

$=\frac{15153\times9}{9}=15153$. Ans.

25. $4\frac{1}{7\frac{1}{5}} \times 3 = 4\frac{5}{36} \times 3 = 12\frac{5}{12}$,

$12\frac{5}{12} + \frac{2}{3} = 12\frac{5+8}{12} = 13\frac{1}{12}$,

$13\frac{1}{12} \times 2\frac{1}{4} = \frac{157}{12} \times \frac{9}{4} = \frac{471}{16}$

$= 29\frac{7}{16}$. Ans.

26. $\frac{3}{5} = \frac{3}{5} \times \frac{36}{36} = \frac{108}{180}$. Ans.

27. $4 = \frac{1}{\frac{1}{4}} = \frac{7}{1\frac{3}{4}}$. Ans.

28. $4\frac{1}{7\frac{1}{2}} = 4\frac{2}{15} = \frac{62}{15} = \frac{62\frac{2}{3}}{5}$

$= \frac{62\frac{2}{3} \times 20}{100} = \frac{413\frac{1}{3}}{100}$. Ans.

29. $\frac{2\frac{4\frac{1}{2}}{12}}{5\frac{2\frac{1}{4}}{12}} = \frac{2\frac{9}{24}}{5\frac{9}{48}} = \frac{\frac{19}{8}}{\frac{83}{16}} = \frac{38}{83}$

$= \frac{\frac{38}{83}}{1} = \frac{\frac{38}{83} \times 100}{100} = \frac{45\frac{65}{83}}{100}$. Ans.

30.

$\frac{4\text{s. }6\text{d.}}{5\text{s. }8\text{d.}} = \frac{54}{68} = \frac{108}{136} = \frac{4 \text{ lbs.}}{5\frac{1}{17} \text{ lbs.}}$

31. Son's share $= 1 - \frac{1}{3} - \frac{5}{12}$

$= \frac{12-4-5}{12} = \frac{3}{12} = \frac{1}{4}$. Ans.

32. $\frac{40 \text{ sq. yds. } 6 \text{ ft}}{13\frac{1}{2} \text{ in.}} = \frac{366 \times 144}{13\frac{1}{2}}$

$= \frac{\overset{122}{\cancel{366}} \times \overset{16}{\cancel{144}} \times 2}{\underset{9}{\cancel{27}}} = \underline{3904}$. Ans.

33. $\frac{4815}{379}$ sec. $= \frac{4815}{379.60.60}$ hrs.

$= \frac{107}{30320}$ hrs. Ans.

34. See Hints, p. 321.

35. He saves of his original income $\frac{7}{30} + \frac{1}{5}$ of $\frac{6}{5} + \frac{1}{4}$ of $\frac{7}{5}$ or $\frac{70+72+105}{300}$, or $\frac{247}{300}$.

One half of this he spends in the 4th year, the other half, viz. $\frac{247}{600}$, he spends at the rate of $\frac{1}{2}\left(1 - \frac{6}{25}\right)$ or $\frac{19}{50}$ per annum, and $\frac{247}{600} \div \frac{19}{50} = \frac{247}{600} \times \frac{50}{19} = \frac{13}{12}$, or 13 months, which is the time it will last.

36. His savings can be found, as in **35**, to be $\frac{247}{300}$

of his original income, and his expenditure in the second year to be $\frac{19}{25}$ of his original income. After the fourth year he spends $\frac{1}{2}$ of $\frac{19}{25} \times \frac{13}{12}$, or $\frac{247}{600}$, but $\frac{247}{600} \div \frac{247}{300} = \frac{1}{2}$. Ans.

37. As in **35**, the savings can be shown to be $\frac{247}{300}$ of the original income, of which, after the 4th year, he has only $\frac{1}{2}$ left, viz. $\frac{247}{600}$; and for this to last him $1\frac{1}{12}$ of a year, his expenditure must be $\frac{247}{600} \times \frac{12}{13}$ or $\frac{19}{50}$ of his original income, and this is $\frac{1}{2}$ of $\frac{19}{25}$ or his expenditure in 2nd year.

38. See Hints, p. 321.

39. $7\frac{1}{61} - 5\frac{3}{5} = 7\frac{7}{43} - 5\frac{3}{5}$

$= 6\frac{50}{43} - 5\frac{3}{5} = 1\frac{250 - 129}{43 \times 5}$

$= 1\frac{121}{215}$ and $4\frac{2}{7}$ of $\frac{7}{15}$

$- 1\frac{121}{215} = \frac{30}{7}$ of $\frac{7}{15} - 1\frac{121}{215}$

$= \frac{94}{215}$. Ans.

40. $\frac{\frac{2}{3}+\frac{3}{4}}{\frac{2}{3}\text{ of }\frac{3}{4}} \div \frac{\frac{3}{4}-\frac{2}{3}}{\frac{3}{4}\div\frac{2}{3}} = \frac{8+9}{\frac{12}{1}}$

$\times \frac{\frac{9}{8}}{\frac{9-8}{12}} = \frac{17}{2} \times \frac{9}{2} = \frac{153}{4} = 38\frac{1}{4}$ Ans.

41. First fraction $= 38\frac{1}{4}$

$\times \frac{\frac{3}{4}-\frac{2}{3}}{\frac{3}{4}\div\frac{2}{3}}$ or $\frac{153}{4} \times \frac{\frac{9-8}{12}}{\frac{9}{8}}$,

or $\frac{17}{2} \times \frac{2}{3}$, or $\frac{17}{6}$. The numerator of 1st fraction

$= \frac{17}{6} \times \frac{2}{3}$ of $\frac{3}{4}$, or $\frac{17}{6} \times \frac{1}{2}$,

or $\frac{17}{12}$, and $\frac{17}{12} - \frac{2}{3} = \frac{17-8}{12}$,

or $\frac{9}{12}$ or $\frac{3}{4}$. Ans.

42. The second fraction

$= \frac{\frac{2}{3}+\frac{3}{4}}{\frac{2}{3}\text{ of }\frac{3}{4}} \div 38\frac{1}{4} = \frac{\frac{8+9}{12}}{\frac{6}{12}} \times \frac{4}{153}$

$= \frac{17}{6} \times \frac{2}{9} = \frac{2}{27}$.

The denominator of the second fraction $= \frac{3}{4} - \frac{2}{3} \div \frac{2}{27}$

$= \frac{9-8}{12} \times \frac{27}{2} = \frac{9}{8}$,

and lastly $\frac{3}{4} \div \frac{9}{8} = \frac{3}{4} \times \frac{8}{9} = \frac{2}{3}$ Ans.

Every step in this question depends upon the fact that as $12 \div 3 = 4$, so also $12 \div 4 = 3$, which shows us how to obtain the 3 from 12 and 4.

43. $\frac{3}{7} - \frac{1}{9} = \frac{27-7}{7 \times 9}$ or $\frac{20}{63}$;

$\therefore$ the fraction $= 2\frac{1}{3\frac{1}{4}} \times \frac{20}{63}$

$= 2\frac{4}{13} \times \frac{20}{63} = \frac{30}{13} \times \frac{20}{63} = \frac{200}{273}$.

Ans.

44. 16 | 456 | 27 | 176 | 176 | 522 | 456 | 2) 55 | 27 (nonary)

G.C.M. 27. $\frac{456}{522} = \frac{16}{18}$ (nonary)

Ans.

45.

10 | 45443 | 4330 | 2143 | 54213 | 45443 | 4330 | 4330 | 1 (senary)

G.C.M. = 2143. $\frac{45443}{54213} = \frac{21}{23}$.

Ans.

46. 416 | 402 | 14 | 620 | 416 | 201 | 14 | 31 | 31 | 12 | 1 (septenary)

G.C.M. = 14; $\therefore$ L.C.M.

$= 416 \div 14 \times 620 = 25 \times 620$

$= 23030$ (septenary).

Ans.

47. 3113) 55143 (15 (senary)

3113

24013

24013

243

15

2143

243

5013 Ans.

48. $3\frac{4}{6} \times 1\frac{2}{5\frac{8}{9}} = 3\frac{4}{6} \times 1\frac{2}{3\frac{4}{5\frac{6}{7}}} = 3\frac{4}{6} \times 1\frac{2}{5\frac{6}{7\frac{1}{9}}}$

$= 3\frac{4}{6} \times 1\frac{2}{3\frac{4}{4\frac{1}{7}}} = 3\frac{4}{5\frac{54}{71}} \times 1\frac{2}{3\frac{28}{41}}$

$= 3\frac{284}{409} \times 1\frac{82}{151} = \frac{1511}{409} \times \frac{233}{151}$

$= \frac{352063}{61759} = 5\frac{43268}{61759}$. Ans.

49. $4\frac{3}{2\frac{1}{4}} \div \frac{2\frac{1}{2} - 1\frac{1}{4}}{2\frac{1}{2} + 1\frac{1}{4}} = 4\frac{4}{3} \times \frac{3\frac{3}{4}}{1\frac{1}{4}}$

$= \frac{16}{3} \times \frac{15}{5} = \frac{16}{3} \times \frac{3}{1} = 16.$

In future editions of the work $4\frac{3}{2\frac{1}{4}}$ will be printed $5\frac{\frac{3}{4}}{2\frac{1}{4}}$; at present it is a mixture of a mixed number and an improper fraction.

50. $5\frac{1}{5} - 4\frac{1}{6} = 1\frac{6-5}{5 \times 6} = 1\frac{1}{30}$,

and $3\frac{1}{7} - 1\frac{1}{30} = 2\frac{30-7}{7 \times 30}$, or $2\frac{23}{210}$ (denary).

210)23·00(·10952 . . . (denary)
21 0

2000
1890

1100
1050

500

·10952 ones (denary)
12

1·31424 twelfths
12

3·77068 (twelfth)²s
12

9·24816 (twelfth)³s
12

2·97792 (twelfth)⁴s.

Ans. 2·1393 . . . (duodenary).

Had we carried out the decimal to more places there would have been 3 (twelfth)⁴s.

CHAPTER VI.

1. 50 miles an hour
1760

88000 yds. an hour
3

6\0)26400\0 ft. an hour

6\0)4400\0 ft. a minute

Ans. $73\frac{1}{3}$ ft. a second.

This could have been found better thus, $\frac{50 \times 1760 \times 3}{60 \times 60}$ feet $= 73\frac{1}{3}$ ft.

2, 3. See Hints, p. 322.

4. Take as your unit $\frac{1}{3 \cdot 5 \cdot 12}$ of the entire work. Then each man does 5 of these units each hour, and each woman 3 of them. 2 men do 10 units in each hour, and 6 women do 18 units in each hour, and there are 180 units to be done; ∴ the 2 men and 6 women must work $\frac{180}{10+18}$ or $\frac{45}{7}$, or $6\frac{3}{7}$ hrs. Ans.

5. *See Hints, p. 322.*

6. At the end of 15 days they have done half of the wall; and this A finishes in 20 days; hence he could have finished it alone in 40 days, and does $\frac{1}{40}$ each day. B only does $\frac{1}{30} - \frac{1}{40}$, or $\frac{4-3}{3 \cdot 4 \cdot 10}$, or $\frac{1}{120}$ each day. He would therefore take 120 days. Ans.

7. See Hints, p. 322.

8. Since A and B drink $\frac{4}{5}$ of what C drinks; ∴ C drinks $\frac{3}{7}$ in 15 days, or $\frac{1}{35}$ each day. But A drinks $\frac{1}{70}$ and $\frac{1}{35} \div \frac{1}{70} = 2$, or C drinks twice as much as A each day.

9. A's daily consumption is $\frac{1}{70}$ of the cask; C's is ∴ $\frac{1}{35}$ of the cask, and B's $= \frac{1}{3}\left(\frac{4}{35} - \frac{3}{70}\right) = \frac{1}{3}$ of $\frac{8-3}{70} = \frac{1}{3}$ of $\frac{1}{14}$ of the cask; ∴ A and B and C together drink $\frac{1}{70} + \frac{1}{42} + \frac{1}{35} = \frac{3+5+6}{7 \cdot 2 \cdot 5 \cdot 3}$ or $\frac{1}{15}$ of the cask; therefore they would be 15 days in finishing it. Ans.

10. A drinks $\frac{1}{70}$ of the cask each day, and C drinks $\frac{1}{35}$ each day; $\therefore$ A and C drink $\frac{1+2}{7 \cdot 5 \cdot 2}$ or $\frac{3}{70}$ each day; $\therefore$ B drinks $\frac{1}{15} - \frac{3}{70}$, or $\frac{14-9}{5 \cdot 3 \cdot 2 \cdot 7}$, or $\frac{1}{42}$; and A's + B's daily consumption is $\frac{1}{70} + \frac{1}{42}$, or $\frac{3+5}{7 \cdot 2 \cdot 5 \cdot 3}$, or $\frac{4}{105}$; and $\frac{1}{35} \times 4 = \frac{4}{105} \times 3$, or 4 times C's consumption = 3 times A's and B's.

11. See Hints, p. 323.

12. Since 1 ox cost £8, 15s.,

1 horse
or 2 oxen
„ 12 sheep
„ 10 pigs
„ 100 fowls
} cost £17, 10s.

		£	s.	d.
3 horses	= 1 × 3	52	10	0
6 oxen	= 2 × 3	52	10	0
1 ox	= 2 × ½	8	15	0
96 sheep	= 12 × 8	140	0	0
4 sheep	= 12 × ⅓	5	16	8
40 pigs	= 10 × 4	70	0	0
50 fowls	= 100 × ½	8	15	0
	Ans.	338	6	8

13. Since 1 pig cost £6, 7s. 6d.,

1 horse
or 2 oxen
„ 12 sheep
„ 10 pigs
„ *100 fowls*
} cost £63, 15s. 0d.

		£	s.	d.
3 horses	or $1 \times 3 =$	191	5	0
6 oxen	„ $2 \times 3 =$	191	5	0
1 ox	„ $2 \times \frac{1}{2} =$	31	17	6
96 sheep	„ $12 \times 8 =$	510	0	0
4 sheep	„ $12 \times \frac{1}{3} =$	21	5	0
40 pigs	„ $10 \times 4 =$	255	0	0
50 fowls	„ $100 \times \frac{1}{2} =$	31	17	6
	Ans.	1232	10	0

14. 1 man earns £9, whilst 1 woman earns £4, and 1 child earns £1; ∴ a man earns as much as 9 children, a woman earns as much as 4 children: 6 men and 10 women and 12 children earn as much as 54 + 40 + 12 or 106 children, and they earn 1s. 8d. a day each, or altogether $£\frac{106 \times 1\frac{2}{3}}{20}$, and $£848 \div £\frac{106 \times 1\frac{2}{3}}{20}$ is $\frac{\overset{8}{\cancel{848}} \times \overset{4}{\cancel{20}} \times 3}{\underset{1}{\cancel{106}} \times \underset{1}{\cancel{5}}}$, or 96 days.

Ans.

15. Let us find how many men earn £54 whilst 10 women earn £40 and 12 children earn £12, if they *altogether earn* £848 (a child's wage being 1s. 8d.) in 96 days.

Since they earn £848 in 96 days they earn $£8\frac{5}{6}$ each day, and $£8\frac{5}{6} \div 1\frac{2}{3}$s., or 106, gives us the number of children that the 6 men, 10 women, and 12 children are equivalent to; and 10 women are equivalent to 40 children. If then we take away 40 + 12 from 106, we have 54 as the equivalent of children for the men. But we cannot find how many men, because there are the same number of men in either case; *e.g.*, if 7 men earned £54, while 12 children earned £12, 1 man earned $£\frac{54}{7}$, or $£7\frac{5}{7}$, and is therefore equivalent to $7\frac{5}{7}$ of a child; and the 7 men would, as before, earn as much as 54 children, and the result would be unaffected. The same remark is applicable to the number of women and children.

16. We can find the £848 very easily. As before, show that the 6 men and 10 women and 12 children are equivalent to 106 children; and this $106 \times 1s.\ 8d. \times 96$ will give us

$$£\frac{106 \times \overset{1}{\cancel{8}} \times \overset{4}{\cancel{96}}}{\underset{1}{\cancel{20}} \times \underset{1}{\cancel{3}}} = £848.$$ Ans.

The 1s. 8d. can also be easily found by dividing £848 by 106×96,

$$\text{or } \frac{\overset{8}{\cancel{848}} \times \overset{5}{\cancel{20}}}{\underset{1}{\cancel{106}} \times \underset{3}{\cancel{96}}}\ s. = 1s.\ 8d.$$ Ans.

17. If we take as our unit the work done by each man in each hour, 25 men in 4 days, working 6 hours a day, do $25 \times 4 \times 6$ units, and 24 men in 5 days will have to work $\frac{\overset{5}{\cancel{25}} \times \cancel{4} \times \cancel{6}}{\cancel{24} \times \cancel{5}}$ hrs. to do as much; viz. 5 hrs. a day.

18. Let us find the 24 men. The question would be, how many men working 5 hrs. a day for 5 days would do as much as 25 men in 4 days working 6 hrs. each; as before the latter men do $25 \times 4 \times 6$ units, and therefore the men who work 5 hrs. a day of 5 days will be $\frac{25 \times 4 \times 6}{5 \times 5}$ or 24. Ans.

19. Let us take as our unit the amount of money divided amongst the boys under 10; ∴ 2 units are divided amongst the boys between 10 and 15, and 3 units amongst the elder boys; therefore a unit $= £\frac{100}{1+2+3}$ or £16, 13s. 4d., and this contains 3s. 4d. 100 times; therefore there are 300 boys.

20. Let us take as our unit what *each* boy under 10 receives; then each boy between 10 and 15 receives 3 units, and each over 15 5 units. Since $\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$; ∴ $\frac{1}{6}$ of the school are less than 10. But there are twice as many boys between 15 and 10 as under 10; therefore they receive 6 times what the youngest receive, and there are 3 times as many boys over 15 as under 10; therefore they receive 15 times as much. If we divide £660 by $15+6+1$ or 22, we find that the youngest boys received £30 between them, and the eldest boys £450. Ans.

21. Working as in **20**, the boys between 10 and 15 receive £30 × 6 or £180, and this ÷ 5 gives 36 as the number of boys between 10 and 15, but they are $\frac{1}{3}$ of the school; ∴ there are 108 boys.

22. Since $\frac{1}{2}$ the school are over 15, the £450 found in 20 would have to be divided by 162, which would give each boy $£\frac{450}{162}$, or $£\frac{50}{18}$, or $£\frac{25}{9}$, or £2, 15s. $6\frac{2}{3}$d.

23. 20 men and 20 children have the same as 40 women, or £60; but 20 men and 80 children have £180 − £60, or £120; ∴ 60 children have £60, or each child has £1. Ans.

24. Let us take as our unit a man's day's work, then there are 40 × 25 or 1000 days' work to be done; at the end of 16 days there are 25 × 16 or 400 days' work done, and 600 left to be done, and 25 + 15 or 40 men could do this in $\frac{600}{40}$ or 15 days.

25. The latter men did in 15 days what the former men would have taken 24 days; we have therefore to find a fraction $= \frac{15}{24}$ whose numerator is less than the denominator by 15. Now $\frac{15}{24} = \frac{15}{15+9}$; to reduce 9 to 15 we must multiply it by $\frac{15}{9}$ or $\frac{5}{3}$; ∴ $\frac{15}{15+9} = \frac{25}{25+15}$ Ans $\frac{25}{}$.

26. In how many days would 25 men do what 25 + 15 or 40 men can do in 15 days. Each man does $\frac{1}{40 \cdot 15}$ of the work each day; ∴ 25 do $\frac{25}{40 \cdot 15}$ or $\frac{1}{24}$, and they must have been engaged for 16 + 24 or 40 days. Ans.

27. Let us take as our unit $\frac{1}{3 \times 4}$ of what it costs to feed 3 men or 4 boys.

If $\frac{1}{3 \times 4}$ of cost of 3 men or 4 boys = 1,

$\frac{1}{4}$ cost of 1 man = 1

$\frac{1}{3}$ cost of 1 boy = 1,

cost of man each week is 4 units, cost of boy each week is 3 units, cost of 3 boys is 9 units; but 9 units = 19s. $2\frac{1}{4}$d; ∴ the unit is 2s. $1\frac{7}{17}$d., and the cost of 51 men for a week is 51 × 4 units, or 2s. $1\frac{7}{17}$d. × 51 × 4, or £21, 14s. 11d. Ans.

28. As in **27**, we can find the unit to be 2s. $1\frac{7}{17}$d., and this is contained in £21, 14s. 11d. 51 × 4 times; and since a man costs 4 units each week, ∴ there must be 51 men.

29. There is scarcely any necessity to adopt a unit at all in the working of this question. But since it is suggested

in the question, I will do so. Let our unit be the weight of

$\frac{1}{11 \times 7}$ of the weight of 11 cub. in. of iron or 7 in. of lead; ∴ 1 cub. in. of iron weighs 7 units, 1 cub. in. of lead weighs 11 units, or 1 cub. in. of iron weighs $\frac{7}{11}$ of a cub. in. of lead. We need not know how many of these units are contained in a ton, as we do not require its value.

$$\frac{£36,\ 17s.\ 11d.}{£15} = \frac{8855}{3600} = \frac{1771}{720},$$

which gives us the number of tons of lead in the block. An equal volume of iron would therefore weigh $\frac{7}{11}$ of $\frac{1771}{720}$ tons, and this at £4 a ton would cost $£\frac{4}{1} \times \frac{7}{11} \times \frac{1771}{720}$ or £6, 5s. $2\frac{2}{3}$d.

30. 100 oranges cost $\frac{1}{3\frac{1}{2}}$ or $\frac{2}{7}$ of $\frac{11}{10}$ of 42 pence, and the receipts for them are $\frac{100}{10} \times 3$,

and $\frac{\overset{10}{\cancel{100}}}{\underset{1}{\cancel{10}}} \times \frac{3}{1} - \frac{2}{\cancel{7}}$ of $\frac{11}{\cancel{10}}$ of $\frac{\overset{63}{\cancel{42}}}{1}$

$30 - 13\frac{1}{5}$, or $16\frac{4}{5}$d. Ans.

31. For 100 *oranges* 30d. are received, and $30 - 16\frac{4}{5}$ is the cost of them; but the entire cost is $\frac{11}{10}$ of 42d. or $\frac{231}{5}$d., and $\frac{231}{5}$ contains $13\frac{1}{5}$

$\frac{\overset{21}{\cancel{231}}}{\underset{1}{\cancel{5}}} \times \frac{\overset{1}{\cancel{5}}}{\underset{2}{\cancel{10}}}$ or $\frac{7}{2}$ times;

∴ there were $100 \times \frac{7}{2}$ or 350 oranges.

32. As before in **30**, the cost of 100 oranges can be shown to be $\frac{2}{7}$ of $\frac{11}{10}$ of $\frac{42}{1}$ or $\frac{66}{5}$d., but these were sold for $\frac{66}{5}$d. $+ \frac{84}{5}$d. or $\frac{150}{5}$d. or 30d.; and 100 oranges for 30d. is the same as 10 for 3d.

33. The receipts for 100 oranges are 30d., and of this $16\frac{4}{5}$ are profits; ∴ they cost $13\frac{1}{5}$. Now $3\frac{1}{2}$ hundreds cost $\frac{66}{5}$d. $\times \frac{7}{2}$ or $\frac{33 \times 7}{5}$d. the cost price of all the oranges; and $\frac{33 \times 7}{5} - 42$ is the cost of carriage, and this is

$$= \frac{231 - 210}{5} = \frac{21}{5} \text{ or } 4\frac{1}{5}\text{d.},$$

that is $\frac{1}{10}$ of 42d.

42. Using the same unit as in **41**, we find there are 22500 units in 155s.; and that therefore each unit is $£\frac{155}{22500 \times 20}$; but £4, 2s. 8d. contains this unit 12000 times, and 15 horses in 8 days cost 9120 units. Hence the sheep cost 2880 units; and this $\div 8 \times 5$ units (the cost of each sheep for 8 days) gives us the 72 sheep. Ans.

43. We cannot here use the unit used in **41**.

horses		sheep		£	s.	d.		days
20	and	196	cost	7	15	0	to keep	9
∴ 5	„	49	„	1	18	9	„	9
∴ 5	„	49	„	0	4	$3\frac{2}{5}$	„	1
∴ 15	„	147	„	0	12	11	„	1
but 15	„	72	„	4	2	8	„	8
∴ 15	„	72	„	0	10	4	„	1
∴		75	„	0	2	7	„	1
∴		1	„	0	0	$0\frac{31}{75}$	„	1
but 5	„	49	„	0	4	$3\frac{2}{5}$	„	1

∴ 5 horses will cost 4s. $3\frac{2}{5}$d. $-\frac{31}{75}$d. $\times 49$, or $31\frac{31}{75}$d.; but $31\frac{31}{75}$d. contains $\frac{31}{75}$d. (the daily keep of a sheep) 76 times. ∴ 5 horses cost as much as 76 sheep. Ans.

44. Taking as our unit $\frac{1}{5 \times 7}$ of what she had, she spent 14 of her units, of which she originally had 35; and $\frac{4}{7}$ of $(35 - 14)$ or 12 units = 1s. 9d.; ∴ each unit is $= \frac{21}{12}$d., and her money was $\frac{21}{12}$d. $\times \frac{35}{1}$ or $\frac{245}{4}$d. or $61\frac{1}{4}$, or 5s. $1\frac{1}{4}$d. Ans.

45. Here we cannot use the unit we used in **44**. Since she originally had 5s. $1\frac{1}{4}$d., and has now left $\frac{7}{4}$ of 21d., or $\frac{147}{4}$d.; she spent at first $\frac{245 - 147}{4}$d. or $\frac{98}{4}$ or $\frac{49}{2}$d., and $\frac{49}{2}$d. is $\frac{\cancel{49}}{\cancel{2}} \times \frac{2}{\cancel{4}_{\cancel{2}}} \frac{\cancel{4}}{\cancel{5}}$ or $\frac{2}{5}$ of 5s. $1\frac{1}{4}$d. Ans.

46. We can solve this question without taking the cost into consideration, thus:—

Let $\frac{1}{3\times5}$ of 3 men's or 5 boys' work each day be one unit; ∴ each man does 5 units a day, and each boy does 3 units a day; ∴ 40 boys in 8 days do $40\times8\times3$ units, and 15 men in 8 days do $15\times8\times5$ units, or in all 1560 units. But 20 boys do 60 units each day, and 20 men do 100 units each day; ∴ $\frac{1560}{60+100}$ or $9\frac{3}{4}$ will be the days that 20 men and 20 boys can do the same work in.

47. Each boy does $\frac{3}{5}$ of a man's work, but only receives $\frac{2}{3}$ of a man's wage; it is therefore more profitable to employ men than boys, since $\frac{3}{5}$ is less an $\frac{2}{3}$. Let us take as our unit $\frac{1}{2\times3}$ of 2 men's or three boys' day's wages. A man's daily wages are three of these units, and a boy's are two; ∴ 40 boys in 8 days receive 640 units, and 15 men in 8 days receive 360 units; and ∴ a unit is $£\frac{35s}{640s+360s}$ or $£\frac{7}{20}$. In the latter case the boys earn $£\frac{7}{20}\times20\times9\frac{3}{4}\times2$, and the men earn $£\frac{7}{20}\times20\times9\frac{3}{4}\times3$, or altogether

$$£\frac{7\times\cancel{20}^{5}\times39\times\cancel{5}}{\cancel{20}_{4}\quad\cancel{4}},\text{ or}$$

£341, 5s. 0d. Ans.

48. This is not easy.

boys		men		days		£
40	and	15	in	8	earn	350
and 20	„	20	„	$\frac{39}{4}$	„	$\frac{1365}{4}$
∴ 40	„	15	„	1	„	$\frac{350}{8}$
and 20	„	20	„	1	„	$\frac{1365 \times 4}{4 \times 39}$
or 40	„	40	„	1	„	$\frac{1365 \times 2}{39}$

Subtracting, we find that 25 men in one day earn $£\frac{280 - 175}{4}$,

or 1 man earns $£\frac{\overset{21}{\cancel{105}}}{4 \times \underset{5}{\cancel{25}}}$, or $£\frac{21}{20}$ each day. Again, since

boys		men			day	£
40	and	15	earn	in	1	$\frac{175}{4}$
$\frac{4 \times 40}{3}$	„	20	„		1	$\frac{175 \times \cancel{4}}{\cancel{4} \times 3}$
but 20	„	20	„		1	$\frac{1365 \times 4}{4 \times 39}$

∴ $53\frac{1}{3}$ − 20 boys in one day earn $£\frac{175 - 105}{3}$; and ∴ 1 boy

earns $£\frac{\overset{7}{\cancel{70}} \times \overset{1}{\cancel{3}}}{\underset{1}{\cancel{3}} \times \underset{2}{\cancel{10}}}$ or $£\frac{7}{10}$, and $£\frac{7}{10}$ is $\frac{\overset{1}{\cancel{7}}}{\underset{3}{\cancel{10}}} \times \frac{\overset{2}{\cancel{20}}}{\underset{3}{\cancel{21}}}$ or $\frac{2}{3}$ of $£\frac{21}{20}$; that

is, a boy's wages are $\frac{2}{3}$ of a man's. Ans.

49. A has walked 2 extra miles, which take him half an hour. One of these he walks by himself in $\frac{60}{4\frac{1}{2}}$ min. or $13\frac{1}{3}$ min. He therefore walks the other (with B), in $30 - 13\frac{1}{3}$, or $16\frac{2}{3}$ min.; therefore B's pace is $\frac{60}{16\frac{2}{3}}$ or $\frac{180}{50}$ or $3\frac{3}{5}$ miles an hour. Ans.

50. After A has left B, he has 9 miles to walk, and this he has to do in the time he would have walked 8 miles less the time he has walked the mile with B. He would have walked the 8 miles in $\frac{8 \times 60}{4\frac{1}{2}}$, or $\frac{8 \times 60 \times 2}{9}$, or $106\frac{2}{3}$ min.; and he took $\frac{60}{3\frac{3}{5}}$ or $\frac{60 \times 5}{18}$ or $16\frac{2}{3}$ min. to walk the mile with B. Therefore he would have to walk 9 miles in $106\frac{2}{3} - 16\frac{2}{3}$ or 90 min.; that is, one mile in 10 min., or six miles an hour. Ans.

CHAPTER VII.

1. £1626, 15s. = £1626·75.

£1626·75	$\frac{15}{16}$ of £16, 2s. 4d.
·0125	
813375	
325350	$= \frac{15}{16}$ of $16\frac{2\frac{1}{3}}{20}$
162675	
£20·334375	
20	$= \frac{15}{16}$ of $16\frac{7}{60}$
6·6875ȣȣs.	
12	
8·25ȣȣd.	$= \frac{15}{16}$ of $\frac{967}{60}$
4	
1·00	$=$ £15, 2s. $2\frac{1}{4}$d.

£20, 6s. $8\frac{1}{4}$d.

∴ the difference is £5, 4s. 6d. Ans., and £1, 6s. $1\frac{1}{2}$d. or £1·30625 contains £5, 4s. 6d. or £5·225 ·25 times. Ans.

2. $\frac{5}{7}$ of a guinea $+\frac{3}{5}$ of a sovereign $+\frac{7}{20}$ of a crown $+\frac{1}{4}$ of a shilling $=\frac{5}{7}$ of $21+\frac{3}{5}$ of $20+\frac{7}{20}$ of $5+\frac{1}{4}$, or $15+12+1\frac{3}{4}+\frac{1}{4}$, or 29s. Ans.

29s. $=£\frac{29}{20}$; and $£\frac{29}{20} \div 4\frac{7}{20}$ of $£1 = \frac{29}{20} \times \frac{20}{87}$ or $\frac{1}{3}$. Ans.

3. $3\frac{5}{8}$ pounds $+9\frac{3}{7}$ guineas $+2\frac{4}{5}$ half-crowns $=\frac{29}{8}$ of $20+\frac{66}{7}$ of $21+\frac{14}{5}$ of $\frac{5}{2}$, or $\frac{145}{2}+198+7$, or $72\frac{1}{2}+198+7$, or $277\frac{1}{2}$ or £13, 17s. 6d.

4. $\frac{\frac{1}{2}+\frac{1}{3}-\frac{1}{6}}{\frac{1}{2}\text{ of }\frac{1}{3}\text{ of }\frac{1}{6}}$ of $£3\frac{4}{20}$

$$= £\frac{\frac{3+2-1}{6}}{\frac{1}{2\times 3\times 6}} \text{ of } \tfrac{16}{5}$$

$= £4 \times 6$ of $\frac{16}{5}$ or $£\frac{384}{5}$ or £76, 16. Ans.

5.

$$\frac{3\frac{1}{7}-\frac{2}{3}}{\frac{4}{49}\times 7\frac{7}{12}} = \frac{2\frac{24-14}{7\times 3}}{\frac{4}{7}\times\frac{91}{12}\ 13}$$

$$= \frac{\frac{5}{1}\ 4}{\frac{1}{1}\times\frac{1}{3}} = 4. \text{ Ans.}$$

.00'6) .000'36

0.06 Ans.

6.

301·28) 24·10'9932 (0·080025
24 1024

75320
60256

150640
150640

Ans ·080025

·90625 cub. yd.
27

634375
181250

24·46875 ft.
1728

375000
93750
328125
46875

810·00000 in.

Ans. 24 cub. ft. 810 in.

7.

$\frac{\cancel{2}}{\cancel{7}}$ of $\frac{\overset{3}{\cancel{22}}}{\cancel{22}\ 11}$ of $£15\frac{10\frac{\cancel{8}}{\cancel{4}}}{20} = £\frac{3}{11}$ of

$£15\frac{43}{80} = £\frac{3}{\cancel{11}}$ of $\frac{\overset{113}{\cancel{339}}}{\cancel{80}} = £\frac{339}{80}$

$\frac{\cancel{7}}{\cancel{13}\cancel{9}\ \cancel{13}}$ of $\frac{\cancel{10}}{\cancel{3}}$ of $\frac{\overset{39}{\cancel{78}}}{\cancel{14}\ 2}$ of

$£2\frac{15\frac{2}{\cancel{5}}}{20} = £\frac{1}{2} \times 2\frac{47}{60} = £\frac{1}{2}$

$\times \frac{167}{60} = £\frac{167}{120}$ and $£\frac{339}{80}$

$- £\frac{167}{120} = £\frac{1017 - 334}{40.2.3}$

$= £\frac{683}{40.2.3} = £2,\ 16s.\ 11d.$

$\frac{683}{40.2.3}$ is $\frac{683}{40.2.3} \div \frac{1}{8}$ of half

a crown, and $\frac{683}{\cancel{40}.2.3} \times \frac{\overset{8}{\cancel{8}}}{1}$
5

$= 22\frac{23}{30}$ Ans.

8. $\frac{6}{11} + 6\frac{1}{11} + \frac{6}{11}$ of $\frac{67}{11}$

$= 6\frac{66 + 11 + 402}{11 \times 11} = 6\frac{479}{121}$

$= 9\frac{116}{121}$ Ans.

3 qts. 1 pt. is $\frac{3 \text{ qts. } 1 \text{ pt.}}{3 \text{ bus. } 1 \text{ pk.}}$

of 3 bus. 1 pk. and $\frac{3 \text{ qts. } 1 \text{ pt.}}{3 \text{ bus. } 1 \text{ pk.}}$

$= \frac{7 \text{ pts.}}{13 \times 2 \times 4 \times 2 \text{ pts.}}$

$= \frac{7}{208}$ Ans.

9. $\frac{3}{12}$ of $\frac{5}{9}$ of $£5 - \frac{7}{15}$ of a

guinea $= \frac{1}{4}$ of $\frac{5}{9}$ of $\frac{100}{1}$s. $- \frac{7}{15}$ of

21s. $= \frac{125}{9} - \frac{49}{5}$, or $\frac{625 - 441}{5 \cdot 9}$

or $\frac{184}{45}$s., and $\frac{184}{45}$ is $\frac{184}{45} \div 69$

of £3, 9s., and $\frac{\overset{8}{\cancel{184}}}{45} \times \frac{1}{\cancel{69}}$
3

$= \frac{8}{135}$ Ans.

10. $\frac{384}{990}$ of $£8\frac{16\frac{1}{4}}{20} = \frac{\overset{24}{\cancel{384}}}{990}$ of $\frac{141}{\cancel{16}} = £\frac{\overset{4}{\cancel{24}} \times \overset{47}{\cancel{141}}}{\overset{\cancel{990}}{\cancel{330}}} = £\frac{4 \times 47}{55}$

$= £3\frac{23}{55}$.

$6\frac{1}{2}$ of $\frac{16}{65}$ of $£\frac{8\frac{1}{2}}{20} = £\frac{\cancel{13}}{\cancel{2}}$ of $\frac{\overset{8}{\cancel{16}}}{65}$ of $\frac{7\frac{17}{24}}{\underset{5}{20}} = £\frac{8}{5}$ of $\frac{\overset{37}{\cancel{185}}}{\cancel{24} \times 20}$

$= £\frac{37}{60}$, and $\frac{7}{11}$ of 1d.

$= £\frac{7}{240 \times 11}$, and these added together are

$$£3\frac{1104 + 1628 + 7}{5 \cdot 11 \cdot 12 \cdot 4},$$

or $£3\frac{\overset{83}{\cancel{249}}}{\underset{4}{\cancel{2739}}}$, or $£4\frac{3}{80}$, or £4, os. 9d.

11. $\frac{\overset{417}{\cancel{8349}} \times 21}{\cancel{49}}$ or 8757 sovereigns. Ans.

12. $$\frac{(\frac{1}{2}+\frac{2}{3}) \text{ of } (\frac{3}{4}+\frac{4}{5}) + (\frac{1}{2}+\frac{3}{4}) \text{ of } (\frac{2}{3}+\frac{4}{5}) + (\frac{1}{2}+\frac{4}{5}) \text{ of } (\frac{2}{3}+\frac{3}{4})}{\frac{1}{2} \text{ of } (\frac{2}{3}+\frac{3}{4}+\frac{4}{5}) + (\frac{1}{2}+\frac{2}{3}+\frac{3}{4}) \text{ of } \frac{4}{5}}$$

$$= \frac{\frac{7}{6} \text{ of } \frac{31}{20} + \frac{5}{4} \text{ of } \frac{22}{15} + \frac{13}{10} \text{ of } \frac{17}{12}}{\frac{1}{2} \text{ of } \frac{40+45+48}{3 \cdot 4 \cdot 5} + \frac{6+8+9}{2 \cdot 3 \cdot 2} \text{ of } \frac{4}{5}}$$

$$= \frac{\frac{217+220+221}{2 \cdot 3 \cdot 2 \cdot 2 \cdot 5}}{\frac{1}{2} \text{ of } \frac{133}{3 \cdot 4 \cdot 5} + \frac{23}{2 \cdot 3 \cdot 2} \text{ of } \frac{4}{5}}$$

$$= \frac{658}{133+184} = 2\frac{\underset{317}{24}}{\cancel{317}} \text{ Ans.}$$

13.

$$£\left(\frac{342}{990} \text{ of } 9\tfrac{1}{6}+\frac{5}{6} \text{ of } \frac{3}{\cancel{75}\,\cancel{1000}\,4} \text{ of } \cancel{15}+\frac{5}{\cancel{100}\,\cancel{20}\,4} \text{ of } \frac{\cancel{11}\,\cancel{25}}{\cancel{1000}\,\cancel{40}\,8} \text{ of } \frac{5}{\cancel{3}}\right)$$

$$= \frac{19}{\cancel{38}\,\cancel{110}\,10} \text{ of } \frac{\cancel{11}}{\cancel{24}\,12}+\frac{5}{8}+\frac{3}{32}$$

$$= \frac{76+300+45}{2.5.2.2.3.2.2} = £\frac{421}{480}$$

$= 17\text{s. } 6\tfrac{1}{2}\text{d.}$ Ans.

2)1 halfpenny

12)6·5 pence

20)17·5416666 etc. s.

50)·8770833 etc. £

·017541666 Ans.

14. $£\left(\frac{3}{8} \text{ of } 1\frac{1}{2}+\frac{\cancel{2}}{\cancel{3}} \text{ of } \frac{\cancel{3}}{\cancel{2}} \text{ of } \frac{4}{5} \text{ of } 1\frac{1}{4}-\frac{5625}{10000}\right)$

$$= £\left(\frac{9}{16}+1-\frac{5625}{10000}\right)$$

16)9·0000

£·5625, ∴ $£\frac{9}{16}$

$$= £\frac{5625}{10000};$$

∴ the sum = £1 Ans.

16)8 oz.

28)3·5 lbs.

4)3·125 qrs.

20)2·78125 cwts.

·1390625 tons. Ans.

15. $£\left(\frac{1}{4} \text{ of } 17\frac{2}{3}+\frac{2625}{1000} \text{ of } \frac{1}{20}-\frac{3}{4} \text{ of } \frac{5}{8} \text{ of } 5\frac{1}{3}+\frac{237}{900} \text{ of } \frac{5}{4}\right)$

$$= £\left(\frac{53}{4.3.20}+\frac{105}{40.20}-\frac{1}{8}+\frac{237}{180.4}\right)$$

$$= £\frac{1590+945-900+2370}{4.3.20.10.3}$$

$$= £\frac{\overset{2}{\cancel{267}}}{\cancel{4000}\,\cancel{20}.\cancel{3}}$$

$= 11\text{s. } 1\tfrac{1}{2}\text{d.}$ Ans.

2)1 halfpenny

12)1·5d.

20)11·125s.

5) ·55625£

·11125 Ans.

16. $\frac{4}{5}$ ac. $+\frac{7}{36}$ of $\frac{1}{4}$ ac. $+\frac{8}{9}$ of $\frac{1}{4}$ of $\frac{1}{5}$ ac. $=\frac{576+5+4}{5\cdot4\cdot9\cdot4}$ acs.

$=\frac{585}{5\cdot4\cdot9\cdot4}$ ac. $=\frac{13}{16}$ ac.

$=3$ ro. 10 po. Ans.

$£\left(\frac{15}{36}+\frac{56}{20}\right)\sim£\frac{416}{1000}+\frac{83}{20\times100}$

$=£\frac{75+504}{9\cdot4\cdot5}\sim\frac{832+83}{20\times100}$

$=£\frac{57900\sim8235}{9\cdot2\cdot2\cdot5\cdot100}$

$=£\frac{3311}{12\cdot20\cdot5}$, or 3311 fifths of a penny.

5)3311

12)662$\frac{1}{5}$d.

55s. 2$\frac{1}{5}$d.

Ans. £2, 15s. 2$\frac{1}{5}$d.

17. $\frac{\frac{1}{3}\left(\frac{1}{2}+\frac{1}{7}\right)-\frac{1}{7}}{1+\frac{1}{2-\frac{1}{4}}}=\frac{\frac{1}{3}\text{ of }\frac{9}{14}-\frac{1}{7}}{1+\frac{4}{8-1}}$

$=\frac{\frac{3-2}{7\cdot2}}{\frac{7+4}{7}}=\frac{1}{22}$ Ans.

18. See Hints, p. 325.

19. $£\left(\frac{1}{10}\text{ of }\frac{1}{4}+\frac{3}{4}\right)-£\left(\frac{3}{16}\text{ of }\frac{1}{10}+\frac{3}{5}\text{ of }\frac{1}{8}\right)=£\left(\frac{1}{10.4}+\frac{3}{4}\right)-£\left(\frac{3}{8.2.2.5}+\frac{3}{5.8}\right)$

$=£\frac{1+30}{10.4}-£\frac{3+12}{8.2.2.5}$

$=£\frac{31}{10.4}-£\frac{15}{8.2.2.5}$

$=£\frac{124-15}{5.2.2.8}=£\frac{109\times3}{20\times12\times2}$,

or 327 halfpence, or 13s. 7$\frac{1}{2}$d. Ans.

2)1 halfpenny

12)7·5 pence

20)13·625s.

5)·68125

·13625 Ans.

20. $\dfrac{\frac{7}{16}\text{ of }9\frac{17\frac{1}{3}}{20}}{9\frac{1}{4}\text{ of }\frac{21}{20}}=\dfrac{\frac{7}{16}\text{ of }9\frac{52}{3.20}}{\frac{37}{4}\text{ of }\frac{21}{20}}$

$$=\frac{\frac{\cancel{7}}{\cancel{16}}\text{ of }\frac{\cancel{52}}{3\cdot\cancel{20}}}{\frac{\cancel{37}}{\cancel{4}}\text{ of }\frac{\cancel{21}}{\cancel{20}}}=\frac{4}{9}\text{ Ans.}$$

21. £1, 11s. 5½d. contains 755 halfpence, and each egg costs a halfpenny.

Ans. 755 eggs.

22. 12)7·5 in.

3)2·625 ft.

·875 yd.

Ans. ·00875.

23. For what he pays 2½d. he receives $\frac{160}{100}$, or $\frac{8}{5}$ of 2½d. or 4d.; but to receive 4d. he must sell 1⅓ quarts of milk; hence he dilutes with ⅓ of water. *Ans.*

24. $\frac{2}{9}$ of 21s. $+\frac{1}{7}$ of $26\frac{1}{3}$s. $+\frac{1}{5}$ of $3\frac{2}{3}$s. $=\left(\frac{14}{3}+\frac{79}{7\cdot3}+\frac{11}{5\cdot3}\right)$s.

$=\frac{490+395+77}{3\cdot7\cdot5}$ or $\frac{962}{3\cdot7\cdot5}$s.

$=9\text{s. }1\frac{33}{35}\text{d.}$

and $\frac{962}{3\cdot7\cdot5}$ is $\dfrac{962}{6\frac{2}{3}}$ or $\dfrac{\cancel{962}^{481}}{10\cdot\cancel{3}}$

or $\frac{481}{350}$ of 6s. 8d. Ans.

25. $2\frac{1}{2}$ gills is $\dfrac{2\frac{1}{2}}{3\frac{1}{8}\times4\times2\times4}$,

or $\dfrac{5\cdot3}{2\cdot\cancel{10}\cdot4\cdot2\cdot4}$, or $\dfrac{3}{128}$,

or ·0234375 of $3\frac{1}{8}$ gallons. Ans.

26. $£\left(\frac{2}{3}\text{ of }\frac{1}{4}+\frac{13}{\cancel{12}\cdot\cancel{6}}\text{ of }\frac{\cancel{21}}{\cancel{20}}+\frac{1}{3}\right.$

$\left.\text{of }\frac{18\frac{1}{2}}{20}+\frac{15}{36}\right)=£\left(\frac{1}{6}+\frac{13}{6.20}+\frac{1}{3}\right.$

$\left.\text{of }\frac{37}{40}+\frac{5}{12}\right)$

$=£\frac{20+13+37+50}{3.2.2.5.2}$

$=£\frac{120}{3.2.2.\cancel{5}.\cancel{2}}=£1.$ Ans.

27. 500 quarters at 56s. a quarter is $£\frac{\overset{25}{\cancel{500}} \times 56}{\cancel{20}}$, or £1400 which he has to pay.

He receives on the whole £1425. For half he receives $\frac{\overset{4}{\cancel{8}} \times 6 \times 25\cancel{0}}{\cancel{20}}$; therefore for the other half he receives £1425 − 600, or £825, and this contains 250 $3\frac{3}{10}$ times, or the remaining 250 quarters must be sold at £3, 6s. a quarter.

28. £5, 2s. 6d. = £5·125.

£5·125
3·1475

25625
35875
20500
5125
15375

£16·1309375
20

2·61875 00s.
12

7·425 00d.

Ans. £16, 2s. $7\frac{17}{40}$d.

29. $£\left(\frac{3}{5} \text{ of } 1\frac{6\overset{6\frac{3}{4}}{\cancel{12}}}{20} + \frac{11}{\cancel{33}\cancel{6}\ 16} \text{ of } \frac{\cancel{21}}{20} + \frac{5}{64} + \frac{4375}{20 \times 10000} + \frac{2}{15} \text{ of } \frac{1}{2}\right)$

$= £\left(\frac{3}{5} \text{ of } 1\frac{6\frac{9}{16}}{20} + \frac{11}{16 \cdot 20} + \frac{5}{64} + \frac{175}{20 \times 400} + \frac{1}{15}\right) = £\left(\frac{3}{5} \text{ of } 1\frac{\overset{21}{\cancel{105}}}{16 \cdot \overset{4}{\cancel{20}}} + \frac{11}{16 \cdot 20} + \frac{5}{64} + \frac{7}{20 \times 16} + \frac{1}{15}\right)$

$= £\frac{3 \cdot 85 \cdot 3 + 33 + 75 + 21 + 64}{5 \cdot 4 \cdot 16 \cdot 3}$

$= £\frac{958}{20 \cdot 12 \cdot 4}$, or 958 farthings,

and this = 19s. $11\frac{1}{2}$d.

30. $2\frac{1}{8}$ of $£5\frac{11}{20}$ is $\frac{2\frac{1}{8} \text{ of } 5\frac{11}{20}}{2\frac{5}{16} \text{ of } 4\frac{1}{4}}$,

or $\frac{\frac{\cancel{17}}{8} \text{ of } \frac{\overset{3}{\cancel{111}}}{\cancel{20}}}{\frac{\cancel{37}}{\cancel{16}} \text{ of } \frac{\cancel{17}}{\cancel{4}}}$, or $\frac{\overset{\cancel{2}}{\cancel{6}}}{\overset{2}{\cancel{5}}}$ of $2\frac{5}{16}$ of £4, 5s.

Ans. $\frac{6}{5}$, or $1\frac{1}{5}$.

31. In the six coins there are $\frac{2}{5}+\frac{6}{11}+\frac{21}{19}$ of gold,

or $\frac{418+570+1155}{5\times11\times19}$, or $\frac{2143}{1045}$;

and in 6 coins there are 6270 parts, or the gold is $\frac{2143}{4127}$ of the silver. Ans.

32. $7\frac{3}{7}-\frac{3}{7}$ of $7\div7\cdot3$ is $\frac{7\frac{3}{7}-3}{7\frac{3}{10}}$,

or $\frac{\frac{31}{7}}{7\frac{3}{10}}$, or $\frac{310}{511}$ of £7·3. Ans.

33. $\frac{\cancel{24}}{10.2.\cancel{24}.60.60}$, or $\frac{1}{2.60.60.10}$; ∴ the faster clock travels $1\frac{1}{72000}$ secs., whilst the slower and accurate clock goes 1 sec. Ans.

34. 89 gals. 1 qt. 1 pt. is

$\frac{\cdot357\frac{1}{2}}{572\times4}$, or $\frac{\overset{5}{\cancel{65}}}{\underset{5\cancel{2}}{\cancel{715}}\cancel{2}.4}$, or $\frac{5}{2.4.4}$,

or ·15625 of 572 gals. Ans.

35. $\frac{\frac{7}{16}\text{ of }\frac{9}{11}}{2\frac{1}{3}\text{ of }1\frac{1}{4}}=\frac{\frac{\cancel{7}}{\cancel{16}}\text{ of }\frac{9}{11}}{\frac{\cancel{7}}{3}\text{ of }\frac{5}{\cancel{4}}}$

$=\frac{9.3}{4.11.5}$, or $\underline{\frac{27}{220}}$ Ans.

$\frac{27}{220}$ of 1 ac.

$=\frac{\overset{2}{27}\times\cancel{4}\times\cancel{40}\times\overset{11}{\cancel{121}}}{\cancel{121}\times\cancel{40}}$ yds.

$=\underline{594\text{ sq. yds.}}$ Ans.

36. 8)1·

8)·125

·015625 Ans.

$\frac{22}{3125}=\frac{176}{25000}=\frac{704}{100000}$

$=\underline{\cdot00704}$ Ans.

·015625)·00704000(0·45056

62500

79000

78125

87500

78125

93750

93750

· · · ·

Ans. ·45056.

$\frac{45056}{100000}=\frac{5632}{12500}=\frac{1408}{3125}$. Ans.

37. Since 1 litre = ·22 gal., 100 litre = 22 gals., or 22.4.2 pts., and 100 litre cost $£\frac{1000 \times 49}{1200}$; ∴ the cost of 22.4.2 pts. is $£\frac{5 \times 49}{6}$, and the cost of 1 pint is $£\frac{5 \times 49}{6.22.4.2}$, or $£\frac{245}{1056}$, which is nearly 4s. 8d. Ans.

38. A troy dwt. = 24 grs.; an avoir. oz. = $\frac{7000}{16}$ grs. The L. C. M. of 24 and $\frac{7000}{16}$ is 3.2.2.2.7.5.5.5 grs., or 7·5·5·5 dwts., which are 3 lbs. 7 oz. 15 dwts.

39.
12)9·9 in.
3)·825 ft.
5½)·275 yds.
40)·05 poles
8)·00125
·00015625 Ans.

40. $\frac{15\frac{3}{5}}{7\frac{4}{5}}$ of £1 + $\frac{1}{5}$ of $140\frac{\frac{1}{2}}{20}$ + $\frac{3}{5}$ of $\frac{21}{20}$

$= £\left(\frac{78}{39} + \frac{1}{5} \text{ of } 140\frac{21}{40} + \frac{63}{5 \cdot 20}\right)$

$= £\left(2 + \frac{1}{5} \text{ of } \frac{5621}{40} + \frac{63}{5 \cdot 20}\right)$

$= £\left(2 + 28 + \frac{21 + 126}{5 \cdot 40}\right)$

$= £30\frac{147}{200}$,

or £30, 14s. $8\frac{2}{5}$d. Ans.

41. Since the boat goes from A to C in 7 hrs., it goes from A to B in 3 hrs. 30′; it therefore goes from B to A in 5 hrs. 15′ − 3 hr. 30′, or 1 hr. 45′. It would therefore take twice this, or 3 hrs. 30′ to go from C to A.

42. The difference between 5d. too much and 10d. too little is 15d.; but the difference in price between nuts at 40 a penny and 50 a penny is $\left(\frac{1}{40} - \frac{1}{50}\right)$d. for each nut, or $\frac{5-4}{4.10.5}$, or $\frac{1}{200}$. I therefore buy 15 × 200, or 3000 nuts. 3000 nuts at 50 a penny would cost 60d., but this is 10d. less than I have; therefore the money I had was 5s. 10d. Ans.

43.

$$\text{cwts.} \left(\frac{1}{84} \text{ of } \frac{5}{2} \sim \frac{1}{30} \text{ of } 1\frac{3}{112} \right)$$

$$500 \left\{ \frac{5}{84.2} \sim \frac{\overset{23}{\cancel{115}}}{\underset{6}{\cancel{30}}.112} \right\} \text{ cwts.}$$

$$= 500 \times \frac{20 \sim 23}{7.2.6.2.2.2} \text{ cwts.}$$

$$= \frac{5.\cancel{2}.\cancel{2}.5.5.\cancel{3}}{7.2.\cancel{6}.2.2.\cancel{2}} \text{ cwts.}$$

= 2 cwts. 0 qrs. 26 lbs. Ans.

44.

$$5{\cdot}137 \overline{)147{\cdot}000(28}$$

$$\begin{array}{r} 102\,74 \\ \hline 44260 \\ 41096 \\ \hline 3164 \\ 20 \\ \hline 63280 \\ 61644 \\ \hline 1636 \\ 12 \\ \hline 19632 \\ 15411 \\ \hline 4221 \end{array}$$

Ans. £28, 12s. $3\frac{4\,2\,2\,2}{5\,1\,3\,7}$d., or nearly £28, 12s. $3\frac{3}{4}$d.

45. After he has given away $\frac{1}{8} + \frac{1}{10}$, or $\frac{5+4}{2.4.5}$, or $\frac{9}{40}$, he has $\frac{31}{40}$ left; $\frac{1}{40}$ of his income is therefore $\frac{1}{31}$ of £473, 13s. 1d., and his whole income is $\frac{40}{31}$ of $£473\frac{13\frac{1}{12}}{20}$,

$$\text{or } £\left(\frac{40}{31} \text{ of } 473\frac{157}{12.20} \right),$$

$$\text{or } £\frac{\overset{8}{\cancel{40}}}{31} \text{ of } \frac{\overset{\overset{2}{\cancel{4}}}{\cancel{157}}.\overset{3667}{\cancel{367}}}{\underset{6}{\cancel{12}}.\cancel{20}},$$

£611, 3s. 4d. Ans.

46. 10s. + 5s. + $2\frac{1}{2}$s. + 1s. + $\frac{1}{2} + \frac{1}{3} = 19\frac{1}{3}$, and this is contained in £29.

$$\frac{\overset{29}{\cancel{58}} \times 20 \times 3}{\underset{2}{\cancel{58}}} = 30 \text{ times;}$$

∴ there would be 30 coins of each. Ans.

47. $\left(\frac{5}{7}\text{ of }64\frac{3\frac{3}{4}}{12}+\frac{1875}{10000}\text{ of }111\frac{2}{3}+\frac{2}{3}\text{ of }\frac{9}{11}\text{ of }4\frac{9\frac{3}{4}}{12}+\frac{1}{1000}\text{ of }20\frac{5}{6}\right)$s.

$=\left(\frac{5}{7}\text{ of }64\frac{5}{16}+\frac{75}{400}\text{ of }\frac{335}{3}+\frac{6}{11}\text{of}4\frac{13}{16}+\frac{1}{1000}\text{of}\frac{125}{6}\right)$s.

$=\left(\frac{5}{7}\text{of}\frac{1029}{16}+\frac{3}{16}\text{ of }\frac{335}{3}+\frac{6}{11}\text{ of }\frac{77}{16}+\frac{1}{8\times6}\right)$s.

$=\left(\frac{5\times147}{16}+\frac{335}{16}+\frac{21}{8}+\frac{1}{8\times6}\right)$s.

$=45+20\frac{45+45+126+1}{16\cdot3}$s.

$=\left(45+24\frac{25}{48}\right)$s., or

£3, 9s. $6\frac{1}{4}$d. Ans.

4)1 far.

12)6·25d.

20)9·5208333s.

5)3·476041666£

·695208333 etc. Ans.

48. A gallon weighs $277\cdot274\times\frac{1000}{1728}$ oz., and a ton contains these oz. $\frac{20\times4\times28\times16\times1000\times1728}{277274\times1000}$ times.

$$\frac{138637}{138637)30965760\cdot(223\cdot358\ldots}$$

277274

323836

277274

465620

415911

497090

415911

811790

693185

1186050

Ans. 223·358 . . . gals. in a ton.

Since a gallon contains 277·274 cub. in., a pint contains $\frac{277\cdot274}{4\times2}$ cub. in., or $\frac{277\cdot274}{4\times2\times1728}$ cub. ft., and this weighs $\frac{138\cdot637}{\cancel{277\cdot274}\times\cancel{1000}}\times\frac{125}{\cancel{4}\times\cancel{2}\times\cancel{1728}}$ oz., or $\frac{125}{864}$ oz., or 20·05743 oz. Ans.

49. 8·71875 of 8d.

8

69·75000d.

4

3·00 far.

1·46875 of 6s. 8d.

80

117·50000d.

4

2·0 far.

·0625 of 8 guineas

8

·5000 guineas

21

10·5s.

12

6·0d.

	s.	d.
	5	$9\frac{3}{4}$
Adding,	9	$9\frac{1}{2}$
	15	$7\frac{1}{4}$
Subtracting	10	6
	5	$1\frac{1}{4}$ Ans.

50. $£\left(\frac{3}{4}\text{ of }\frac{1}{9\frac{1}{2}}\text{ of }\frac{38}{20}+\frac{2}{3}\text{ of }\frac{375}{1000}\text{ of }\frac{3}{4}+\frac{2}{5}\text{ of }\frac{425}{990}\text{ of }\frac{8\frac{1}{4}}{20}\right)$

$=£\left(\frac{\cancel{3}}{\cancel{4}_{2}}\text{ of }\frac{\cancel{2}}{\cancel{19}}\text{ of }\frac{\cancel{19}}{10}+\frac{\cancel{2}}{\cancel{3}}\text{ of }\frac{\overset{3}{\cancel{15}}}{\underset{8}{\cancel{40}}}\text{ of }\frac{\cancel{3}}{\cancel{4}_{2}}+\frac{\cancel{2}}{\cancel{5}}\text{ of }\frac{\overset{17}{\cancel{85}}}{\underset{6}{\cancel{198}}}\text{ of }\frac{\cancel{33}}{\cancel{4}.\underset{2}{\cancel{20}}}\right)$

$=£\frac{36+45+17}{2.10.2.2.3}=£\frac{98}{20.12}$,

or 98d. or 8s. 2d. Ans.

CHAPTER VIII.

1. $\frac{64}{3}$ of £384 = £8192 Ans.

£$\frac{7}{12}$ of $\frac{64}{3}$ of $\frac{384}{1}$ = £$\frac{14336}{3}$
= £4778, 13s. 4d. Ans.

2. $\frac{100}{103}$ of £1000 or £$\frac{100000}{103}$

or

£	£	s.	d.
103)100000(	970	17	$5\frac{73}{103}$
927			
730			
721			
90			
20			
1800			
103			
770			
721			
49			
12			
588			
515			
73			

Ans. £970, 17s. $5\frac{73}{103}$.

3. The creditor first receives $\frac{37}{60}$ of his debt, and then $\frac{3}{16}$ of $\left(1-\frac{37}{60}\right)$ or $\frac{592+3\times23}{60\times16}$. He therefore lost $\left(1-\frac{661}{60\times16}\right)$ of £592, or £$\frac{299}{60\times16}\times\frac{37}{1}$, or £184, 7s. 8d. Ans.

4. The chances of the inferior horse are 3 out of 8, and those of the superior 5. Therefore the betting on the better is 5 to 3. Ans.

5. A's play is $\frac{100}{60}$ or $\frac{5}{3}$ of B's; B's is also $\frac{5}{3}$ of C; ∴ A's is $\frac{25}{9}$ or $\frac{1000}{360}$ of C's, or he can give C 640 points in 1000. Ans.

6. A has $\frac{3}{2}$ of B's and $\frac{4}{3}$ of C; ∴ $\frac{4}{3}$ of C is $\frac{3}{2}$ of B's, or C has $\frac{3}{4}$ of $\frac{3}{2}$ of B, or $\frac{9}{8}$ of B's. Ans.

7. 9 to 14 on him. Ans.

8. The chance of the first is $\frac{1}{3}$, that of the second is $\frac{3}{10}$; $\therefore$ on the third it is $1-\frac{1}{3}-\frac{3}{10}$, or $\frac{30-10-9}{30}$ or $\frac{11}{30}$; *i.e.* 11 to 19 on it, or 19 to 11 against it. Ans.

9. If I have $\frac{4}{3}$ of my brother's property he has $\frac{3}{4}$ of mine, or $\frac{1}{4}$ less than I have. Ans.

10. $2(18+10)=56$. Ans.

11. A has $\frac{4}{3}$ of B's; B $\frac{5}{4}$ of C; $\therefore$ A has $\frac{4}{3}$ of $\frac{5}{4}$ or $\frac{5}{3}$ of C, and C has $\frac{3}{5}$ of A's. Ans.

12. A has $\frac{3}{2}$ of B and C's, and B and C have $\frac{6+5}{5}$ of C's; $\therefore$ A has $\frac{3}{2}$ of $\frac{11}{5}$ of C, and C has $\frac{10}{33}$ of A's. Ans.

13. The difference between $\frac{1}{2}$ and $\frac{19}{40}$ is $\frac{1}{40}$; but this $\frac{1}{40}$ is $300-100-80$ or 120 votes; $\therefore$ the number of votes are 40×120 or 4800. A $\therefore$ polls $\frac{4800}{2}-300$, and B $\frac{19\times4800}{40}-100$; that is, A polls 2100 and B 2180. Ans.

A figure will make this quite plain. Let AB represent half the votes, AC $\frac{19}{40}$ of them. CB $\frac{1}{40}$ of the votes. Let BD represent 300 votes, and CE 100. Then DE represents the majority won by A, or 80 votes; but CB $\left(\frac{1}{40}\right)$ = BD − CE − DE, or $300-100-80$.

14.

$\frac{1}{3}$ flows through A each min.

$\frac{1}{5}$ „ B „

$\frac{1}{7\frac{1}{2}}$ or $\frac{2}{15}$ „ C „

$\frac{1}{3}+\frac{1}{5}-\frac{2}{15}=\frac{5+3-2}{15}=\frac{6}{15}$;

$\therefore$ $\frac{2}{5}$ is left in at the end of each minute, and $\therefore$ it will take $\frac{5}{2}$ or $2\frac{1}{2}$ min. to fill it.

15. Let our unit be $\frac{1}{2.3}$ of what 2 men or 3 boys can do each day; then each man does 3 units, and each boy 2 units. 5 men in 17 days can do 255 units, 9 boys in 17 days can do 306 units; or altogether there are 561 units to do. 9 men can do 27 units, and 12 boys do 24 units; ∴ 9 men and 12 boys do 51 units each day, and will take 11 days to do 561 units; hence answer, 11 days.

16, 17. See Hints, p. 326.

18. This is a difficult problem, and will require a good deal of attention to thoroughly grasp. Draw a straight line PQ to represent the course, and mark R near Q. Let RQ = 80. B makes up 80 − 26 yds. under the altered conditions. Now to make up 54 yds. in a certain distance against an opponent, who usually goes faster than you do, is the same as going 54 yds. more than your opponent in the same time, supposing he had been previously going at the same rate as you did. If A had gone at the same rate as B, they would have been together at R on the previous day.

Let us then suppose that A did go at the same rate on the previous day. Whilst A on the second day was going $\frac{8}{9}$ of PR, B went $\frac{9}{10}$ of PR. Mark S and T so that PS = $\frac{8}{9}$ and PT = $\frac{9}{10}$ of PR; then ST = $\frac{9}{10} - \frac{8}{9}$ or $\frac{1}{90}$ of PR. Whilst A was going PS ($\frac{8}{9}$ of PR), B was going PS + $\frac{8}{9}$ of 54; ∴ $\frac{8}{9}$ of 54 = $\frac{1}{90}$ of PR, or PR = 4320 and PQ = 4400, or $2\frac{1}{2}$ miles.

An algebraical solution is as follows, but it is difficult to reduce it to a purely arithmetical one.

Let x yds = course;

$$\therefore \frac{\text{A's pace}}{\text{B's pace}} = \frac{x}{x-80},$$

$$\text{but } \frac{\frac{8}{9}\text{ A's pace}}{\frac{9}{10}\text{ B's pace}} = \frac{x}{x-26},$$

$$\text{or } \frac{\text{A's pace}}{\text{B's pace}} = \frac{x}{x-26} \cdot \frac{9}{10} \cdot \frac{9}{8}.$$

Equating,

$$\frac{x}{x-80} = \frac{x}{x-26} \cdot \frac{9}{10} \cdot \frac{9}{8}.$$

$$80(x-26) = 81\,(x-80),$$
$$x = 80\,(81-26)$$
$$= 4400.$$

Here is a published solution of the problem (Cambridge Senior Local Exam. in 1878), but it is no more arithmetic than the one just above.

Time in latter case $= \frac{9}{8} \times$ time in former, and $\therefore$

$$\frac{\text{course} - 80}{\text{B's speed}} = \frac{8}{9} \cdot \frac{\text{course} - 26}{\frac{9}{10}\text{ B's speed}},$$

and course $= 2\frac{1}{2}$ miles.

19. If we take a man's hour's work as our unit, there are $12 \times 8 \times 14$ units to do, and the half of them are $6 \times 8 \times 14$; and 4 men working $10\frac{1}{2}$ hours each day would do 42 units a day, and $\frac{6 \times 8 \times 14}{42}$ or 16 will be the number of days that the 4 would take to finish it. Ans.

20. If one elephant could do it in 20 hours, two elephants would do it in 10 hours, or do $\frac{1}{10}$ each hour; but 2 elephants and a horse do $\frac{7}{50}$ each hour; therefore the 4 horses do $\frac{7}{50} - \frac{1}{10}$, or $\frac{2}{50}$ of the work each hour, and 1 horse $\frac{1}{100}$ each hour, or 1 horse would take 100 hours to finish the work. Ans.

21. The buying price is $\frac{5}{6}$ of £90; $\therefore$ the profit is $\frac{1}{6}$ of £90 or £15. Ans.

22. In the latter case the engine travels $\frac{3}{2}$ as far in the same time, but the larger wheel only makes $\frac{5}{6}$ as many revolutions. Hence the revolutions made by the larger wheel, according to the data, are $\frac{3}{2}$ of $\frac{5}{6}$ of 704, or 880. Ans.

23. Take as your unit $\frac{1}{5 \cdot 9}$ of the amount of work 5 townsmen or 9 countrymen can do in a day. 5 townsmen can do 5.9 units, or 1 townsman can do 9 units; 9 countrymen can do 5.9 units, or 1 countryman can do 5 units. There are $9 \times 15 \times 10$, or 1350 units to do. 10 townsmen and 12 countrymen do $10 \times 9 + 12 \times 5$ units a day, and finish it in $\frac{1350}{90 + 60}$ or 9 days, and this would cost $(10 \times 7\frac{1}{2} \times 9 + 12 \times 4\frac{1}{2} \times 9)$ shillings, or 1161 shillings; and the town workmen, if they worked by themselves, would cost $15 \times 7\frac{1}{2} \times 10$ or 1125 shillings. It would therefore be cheaper to employ the 15 townsmen by 36 shillings or £1, 16s. Ans.

24. By the data, 3 men and 5 boys could finish the work in 12 days, and 4 men and 6 boys could finish the work in 9 days. Equating the number of men: 12 men and 20 boys could finish the work in 3 days, 12 men and 18 boys could finish the work in 3 days: hence the boys are of

no use. (See Hints to 23, p. 345.)

If 3 men would take 12 days, 1 man would take 36 days; ∴ 3 men in 6 days have done $\frac{18}{36}$, 4 men in 3 days have done $\frac{12}{36}$. There are therefore $\frac{6}{36}$ to be done; and for this to be done in 1 day there must be 6 men, or 2 more than the 4 who completed the third in 3 days. Ans.

25. If each had had 9 units at first, at end of first game, A had $9+3+3$ units, and B and C $(9-3)$ units. At end of second game, A had $\frac{2}{3}$ of $9+3+3$ or 10 units, B had $6+5+2$ or 13 units, and C $6-2$ or 4 units; therefore of what they originally had, A has $\frac{10}{9}$, B $\frac{13}{9}$, and C $\frac{4}{9}$. Ans.

26. Let us take as our unit $\frac{1}{15.2.3}$ of the contents of the cask, then the cask contains 90 units. Through the first pipe each minute there flows 3 units, through the second pipe 2 units, and through the discharge pipe 6 units. The water that is in and passes into the cistern is $90+45\times3+45\times2$ or 315 units; and the water that flows out is 45×6 or 270 units; hence there are 45 units, or a half of the cistern left in. Ans.

27. The price of each inch of candle at the first rate is $\frac{9}{8\frac{1}{2}\times6}$d., or $\frac{3}{17}$d., and the price of each inch at the second rate is $\frac{11}{10\frac{1}{4}\times6}$ or $\frac{22}{123}$d., and the difference between $\frac{3}{17}$ and $\frac{22}{123}$ is $\frac{5}{17\times123}$d.; ∴ the first kind are cheaper by $\frac{5}{17\times123}$d. each inch.

28. In 9 lbs. of the mixture for which he receives 18s., he pays 4d. for chicory; therefore the 8 lbs. of coffee is worth $\frac{1}{8}$ of 17s. 8d. or 2s. $2\frac{1}{2}$d. per lb. Ans.

29. $\frac{16}{3}$ sheep equal in price to 1 ox, $\frac{5}{2}$ oxen equal in price to 1 horse; ∴ 1 horse $= £\frac{5}{\cancel{2}}\times\frac{\overset{2}{\cancel{16}}}{\cancel{3}}\times\frac{5}{\cancel{15}_{4}}$ or £50. Ans.

30. $\dot{0}\dot{9}$ of £6, 13s. $4\frac{1}{2}$d. = $\frac{1}{11}$ of £6, 13s. $4\frac{1}{2}$d. = 12s. $1\frac{1}{2}$d., 15s. − 12s. $1\frac{1}{2}$d. = 2s. $10\frac{1}{2}$d., and 2s. $10\frac{1}{2}$d. = $2\frac{7}{8}$s., or £·14375. Ans.

31. Buying price = $\frac{5}{6}$ of marked selling price, ∴ marked selling = $\frac{6}{5}$ of the buying price; but I only receive $\frac{11}{12}$ of the marked selling price, ∴ I receive for books which cost me £300, $\frac{11}{12}$ of $\frac{6}{5}$ of $\frac{300}{1}$, or £330, and make a profit of £30. Ans.

32. B can do twice as much as A, and C can do twice as much as B, or 4 times as much as A; therefore A, B, C, can do 7 times what A could; but they can finish it in 24 days, therefore A could finish it in 7 × 24 or 168 days, and B in 84 days, and C in 42 days. Ans.

33. $\frac{1}{3}+\frac{1}{4}=\frac{4+3}{12}$; ∴ $\frac{5}{12}$ cost him 12s. 6d. each. Of every 12 sheep 4 cost him 38s., 3 cost him 33s., and the remaining 5, $62\frac{1}{2}$s.; therefore every 12 cost him $133\frac{1}{2}$s., and this is contained in £33, 7s. 6d. exactly 5 times; therefore he bought 12 × 5 or 60 sheep. Ans.

34. B spent £$33\frac{1}{3}$ each year too much; therefore A saves £60 − £$33\frac{1}{3}$ or £$26\frac{2}{3}$. But this is $\frac{1}{8}$ of his income; therefore the income is £133, 6s. 8d. Ans.

35. A spends each year $\frac{4}{5}$ of £$133\frac{1}{3}$ or £$\frac{320}{3}$; therefore B spends £$\frac{320+180}{3}$ or £$\frac{500}{3}$, that is, £$33\frac{1}{3}$ more than he has; hence in 3 years he will owe £100. Ans.

36. B receives in the 3 years £400, but he spends £500; whereas A receives £400, and spends £320, that is, B spends £180 more than A in the 3 years, or £60 a year. Ans.

37. See Hints, p. 327.

38. Let $\frac{1}{9}$ of the pudding be our unit, then there were 2 units of flour, 3 of raisins, and 4 of suet. Had there been pounds of each, it would have cost 2 × 3 + 3 × 6 + 4 × 8 or 56d., but 2s. 4d. or 28d. contains 56d. $\frac{1}{2}$ time; therefore there were half these quantities, viz. 1 lb. of flour, $1\frac{1}{2}$ of raisins, and 2 lbs. of suet, which cost 3d., 9d., and 1s. 4d. Ans.

39. $1801 - 1799 = 2$ hrs., and therefore the watch gains 1 hr., and the clock loses 1 hr.; ∴ 1801 hours as marked by the watch are = 1800 hrs., and $\frac{1 \text{ hr.}}{1800} = \frac{60 \times 60 \text{ sec.}}{1800}$, or 2 sec. Ans.

40. £1562, 3s. 4d. is $\frac{2}{3}$ of my property; ∴ I lose £781, 1s. 8d., and my property is $\frac{3}{2}$ of £1562, 3s. 4d., or £2343, 5s. 0d. Ans.

41. The selling price is $\frac{100}{91}$ of £273, or £300.

42. The cost price is $\frac{100}{111}$ of £259 or £233, 6s. 8d.

43. The water is $\frac{2}{5}$ of 360 gals., or 144 gals.

44. If A's pace is $\frac{3}{5}$ of B's, B's is $\frac{5}{3}$ of A's; it will therefore take B $\frac{3}{5}$ of 4 hrs. to walk what A can walk in 4 hrs., or 2 hrs. 24 min. Ans.

45. We must first subtract 10 florins and 3 guineas, or 83s., from £7, 2s., or 142s. This leaves 59s.; ∴ 12 dollars + 4 × $\frac{27}{60}$ dollars = 59s., or $(12 + 2\frac{4}{25})$ dollars = 59s., or 1 dollar = 50d., or 4s. 2d. and a thaler = $\frac{27}{60}$ of 50d., or 2s. 3d. Ans.

46. His gain is $4\frac{1}{3}$s. × 170 − $3\frac{29}{48}$s. × 180,

or $\left(\frac{13 \times 170}{3} - \frac{173 \times \overset{15}{\cancel{180}}}{\underset{4}{\cancel{48}}}\right)$s., or $\frac{52 \times 170 - 45 \times 173}{3 \cdot 4}$s.,

or $\frac{45 \times 170 + 7 \times 170 - 45 \times 170 - 45 \times 3}{3 \cdot 4}$s.,

or $\frac{1190 - 135}{3 \cdot 4}$s., or 1055d., or £4, 7s. 11d. Ans.

47. 1st candidate's chance is $\frac{1}{11}$ of certainty, the 2nd candidate's, $\frac{1}{3}$; therefore the third's chance is $1-\frac{1}{11}-\frac{1}{3}$, or $\frac{33-3-11}{33}$, or $\frac{19}{33}$, or 19 to 14 *on* him. Ans.

48. A's outlay was $\frac{6}{5}$ of B's buying price, and B's buying price is $\frac{12}{13\frac{1}{2}}$ or $\frac{24}{27}$ or $\frac{8}{9}$ of C's outlay, or $3\frac{3}{4}$s.; ∴ A's outlay was $\frac{6}{5}$ of $\frac{8}{9}$ of $\frac{15}{4}$s., or 4s. Ans.

49. With the stream the man goes 12 miles an hour; against it, under the bank, he can go 8 miles an hour; therefore the rate of the stream and $\frac{2}{5}$ of its rate is 4 miles an hour, or $\frac{4}{1\frac{3}{5}}$ or $2\frac{1}{2}$ miles an hour. Ans.

50. $\frac{100}{112}$ of £5, 6s. 8d., or £$\frac{25}{28}$ of $\frac{16}{3}$ or £$\frac{100}{21}$, or £4, 15s. $2\frac{6}{7}$d. Ans.

CHAPTER IX.

1. See Hints, p. 327.

2. $1\cdot\dot{3}\dot{4}=\frac{134-1}{99}=\frac{133}{99}$.

Note.—As practically we should never perform this operation except to multiply or divide, it is unwise to reduce the improper fraction to mixed numbers.

3. $1\cdot\dot{1}\dot{5}\div 5\cdot\dot{7}=\frac{\overset{2}{\cancel{115}}-1}{99}\times\frac{10}{\cancel{57}}$

$=\frac{20}{99}=\cdot\dot{2}\dot{0}$.

4. $1\cdot\dot{1}\dot{1}\times\cdot\dot{2}\dot{3}=\frac{\overset{37}{\cancel{111}}}{100}\times\frac{\overset{7}{\cancel{23}}-\cancel{2}}{\underset{10}{\cancel{99}}}$

$=\frac{259}{1000}=\cdot 259$.

5. $1\cdot0\dot{6} - 1\cdot06 = \frac{106-10}{90}$

$-\frac{106}{100} = \frac{960-954}{900} = \frac{6}{900}$, or

$\frac{2}{300}$. Ans.

Or better thus:—

$1\cdot0\dot{6} = 1\cdot06666$

$1\cdot06 = 1\cdot06$

difference $= \cdot00\dot{6}$

or $\frac{6}{900}$, or $\frac{2}{300}$. Ans.

6. $2\cdot1\dot{3} \div \cdot42\dot{6} = \frac{213-21}{90}$

$\times \frac{900}{420-42}$, or $\frac{\cancel{192}}{\cancel{90}} \times \frac{\cancel{900}^{10}}{\cancel{384}_{2}} = 5$.

Ans.

7. See Hints, p. 328.

8. Let $F = \cdot343434$, etc.

$100\,F = 34\cdot3434$, etc.

$(100-1)\,F = 34$

$\therefore \cdot\dot{3}\dot{4} = \frac{34}{99}$ (septenary).

9.

$4\cdot7\dot{1}\dot{2}$ (octonary) $= \frac{4712-47}{770}$,

or $\frac{4643}{770}$ *(octonary).* Ans.

10. $5\cdot\dot{7}$ (nonary) $= \frac{52}{8}$ and

$\cdot2\dot{3}$ (nonary) $= \frac{21}{80}$.

$\frac{52}{8} \times \frac{80}{21} = \frac{520}{21}$ (nonary).

$21)520\cdot00(24\cdot\dot{2}3275180\dot{4}$

42

100

84

50

42

70

63

60

42

170

157

120

115

40

21

180

178

100

84

5

Ans. $24\cdot\dot{2}3275180\dot{4}$.

11. $3\cdot1\dot{7}$ of £2, 6s. 3d.

$$= £\frac{317-31}{90} \text{ of } 46\tfrac{1}{4} = \frac{\overset{143}{\cancel{286}}}{\cancel{90}\ 18}$$

of $\frac{\overset{37}{\cancel{185}}}{\cancel{4}\ 2}$s., or $\frac{5291}{18\times 2}$s.

$$36\begin{cases} 4)5291 \\ \hline 9)1322\frac{3}{4} \\ \hline 146\frac{35}{36}\text{s.}, \end{cases}$$

or £7, 6s. $11\frac{2}{3}$d.

12. $\cdot4\dot{9} = \frac{49-4}{90} = \frac{1}{2}$.

£5, 19s. 2d. $\div \frac{1}{2}$
= £11, 18s. 4d. Ans.

13.

$3\cdot1\dot{7}4$ (octonary) $= \frac{3174-317}{700}$,

or $\frac{2655}{700}$ (octonary), or $\frac{1453}{448}$ (denary).

2655	700
8	8
22	56
8	8
181	448 ones
8	
1453 ones	

448)1453(3·24 etc.
1344 See Hints, p. 328.

1090
896

1940
1792

1480

Since $448 = 7 \times 64$ or 7×2^6, the equivalent decimal would be more easily found by short division, dividing by the 7 last.

14, 15, 16. See Hints, p. 328.

17.

13)4d.

22)6·3s.

£2·2$\dot{7}\dot{5}$ (nonary).
Ans.

18. £3·1412 (quinary)
40

12·203$\dot{0}$s.
22

411
411

10·021d.

£3, 12s. 10·021d. (quinary),
or £3, 7s. 5·088d. (denary).

19. 2)1·0

12)3·5 pence

20) ·2916666

·014583

Ans. £·014583.

20.

200
1·05
210·00
1·05
1050
210
220·50
1·05
11025
2205
231·525
1·05
1157625
231525
£243·10125
20
2·025000s.
12
0·300000d.

Ans. £243, 2s. $0\frac{3}{10}$d.

21.

300
1·04
312·00
1·04
1248
312
324·48
1·04
129792
32448
£337·45920
20
9·184000s.
12
2·208000d.

Ans. £337, 9s. $2\frac{26}{125}$d.

2.

180
·98
1440
1620
176·48
·98
14112
15876
172·872
·98
1382976
1555848
169·41456
·98
135531648
152473104
£166·0262688
20
·525376s.
12
6·304512d.

s. £166, os. 6·304512d.

24. *See Hints, p. 328.*

25. $11\dot{2} = \frac{112-11}{9}$, or $\frac{101}{9}$.

$\cdot 3\dot{7} = \frac{37-3}{90}$, or $\frac{34}{90}$, or $\frac{17}{45}$.

$\frac{17}{45})\frac{101 \times 5}{45}($

or $17)505(29$
34
165
153
12

Ans. 29 whole times, with a remainder of 12 forty-fifths, or 4 fifteenths.

26. 3·1675 tons (octonary)
24 (20 denary)
7364
3572
4·53048 cwts.
4
2·5428 qrs.
34 (28 denary)
2610
2046
23·2748 lbs.
20 (16 denary)
5·68 oz.
20 (16 denary)
14·0 drs.

tons	cwt.	qrs.	lbs.	oz.	drs.
Ans. 3	4	2	23	5	14

Expressed in denary the lbs. are 19 and the drams 12.

27. See Hints, p. 328.

28. My profits will be $\cdot 0\dot{3}$ of the buying price, and this on £1000 is $\frac{3}{90}$, or $\frac{1}{30}$ of £1000, or £33, 6s. 8d. Ans.

29. My buying price is $\frac{87-8}{90}$ or $\frac{79}{90}$ of my selling price; ∴ the buying price of £1000 is $\frac{79}{90}$ of £1000.

	£	s.	d.
9)79	00	0	0
877	15	$6\frac{2}{3}$	

∴ my profits are £122, 4s. $5\frac{1}{3}$d. Ans.

30. See Hints, p. 329.

31. My profits are $\cdot 1\dot{6}$ of £112, 10s., or $\frac{15}{90}$ or $\frac{1}{6}$ of £112, 10s., or £18, 15s. Ans.

32. $1 - \cdot\dot{3} - \cdot\dot{3} - \cdot 1 = 1 - \frac{7}{9}$ or $\frac{2}{9}$, but this $\frac{2}{9}$ is £1000; ∴ my gross receipts are $\frac{9}{2}$ of £1000, or £4500. Ans.

33. £4500 − 1000 − $\frac{1}{3}$ of 4500 (wages) − $\frac{1}{9}$ of £4500 (taxes, etc.) = £4500 − £3000, which is what the raw material *costs me*, and £1500 is $\frac{1}{3}$ of £4500, or $\cdot\dot{3}$. Ans.

34. If $\cdot 0\dot{1}$ or $\frac{1}{90}$ of my property is £4, 2s. 6d., the property is 90 times £4, 2s. 6d., or £371, 5s. Ans.

35. If $\frac{125}{10000}$ or $\frac{1}{800}$ of the money is £125, 17s. 6d., or £125$\frac{7}{8}$; the money invested is £$\frac{1007}{8} \times \frac{800}{1}$,

or £100700. Ans.

36. $\frac{£15\ 12s.\ 6d.}{12500} = \frac{15\frac{5}{8} \times 8}{100000}$ $= \cdot 00125$. Ans.

37. The widow receives $\frac{27}{99}$ or $\frac{3}{11}$, the three daughters $\frac{3 \times 142857}{999999}$ or $\frac{3}{7}$ of the estate. The two sons therefore have between them $1 - \frac{3}{11} - \frac{3}{7}$, or $\frac{77-21-33}{77}$, or $\frac{23}{77}$ of £1663$\frac{1}{3}$. But of this the younger son has $\frac{1}{3}$, or £$\left(\frac{1}{3}$ of $\frac{23}{77}$ of $\frac{\overset{36}{\cancel{108}}}{\cancel{385}} \times \frac{\cancel{385}}{5}\right)$, or £$\frac{828}{5}$, or £165, 12s. Ans.

38. Calling C's share 1, B has 2, and A 4; ∴ C has $\frac{1}{7}$ of the whole.

$$7)£376\tfrac{5}{9}$$

or £53 15s. $10\frac{10}{21}$d.

B has £107, 11s. $8\frac{20}{21}$d.

A has £215, 3s. $5\frac{10}{21}$d. Ans.

39.
$$\frac{3\cdot\dot{6}\times\cdot\dot{3}\dot{6}}{\cdot0\dot{3}\dot{6}}=\frac{\frac{33}{9}\times\frac{36}{99}}{\frac{33}{999}}$$
$$100$$

$$=\frac{3600}{99}=36\cdot\dot{3}\dot{6}.$$

40.
$$\frac{12\cdot14\dot{6}\times20}{3\cdot6}$$

$$=\frac{(12146-1214)\,\frac{20}{99}}{\frac{9999\,5}{36-3}}\text{ times}$$
$$9$$

$$=\frac{10932}{5\cdot3\cdot11}\qquad 5)10932$$

$$3)2186\cdot4$$

$$11)728\cdot8$$

$$66\cdot2\dot{5}\dot{4}\text{ Ans.}$$

41. 35 octonary = 29 (denary), $\cdot1\dot{7}$ octonary $=\frac{17-1}{70}=\frac{16}{70}=\frac{14}{56}$ (denary) $=\cdot25$;

∴ $35\cdot1\dot{7}$ (octonary) $=29\cdot25$ (denary). Ans.

42. $4\cdot\dot{7}$ (nonary) $=\frac{47-4}{8}$

$\cdot00\dot{3}$ (nonary) $=\frac{3}{800}$

$$100$$
$$\frac{43}{8}\times\frac{800}{3}=\frac{4300}{3}$$

$=1400$ (nonary). Ans.

43. $2\cdot\dot{5}\div\cdot4\dot{7}=\frac{23}{9}\times\frac{90}{43}$

$\frac{230}{43}=\frac{346}{53}$ (octonary).

$53)346\cdot(5\cdot2624$, etc. Ans.

327

170

126

420

402

160

126

320

254

44. £5·7314 (nonary)

22

15628
15628

17·3018
13

10056
3018

4·0246

Ans. £5, 17s. 4·0246 nonary.

The 17 (octonary)s are 16 (denary)s.

45. 13)8·oood. (nonary)

22)6·6s.

3)2·3£

·7 three £s.

Ans. ·7 (nonary) of £3.

46. $\frac{37145 - 371}{ee0} = \frac{36994}{ee0} = \frac{t854}{2e9}$.

We saw at once that this fraction was reducible by dividing by 4, since the last figure of either number were divisible by 4. Reducing $2e9$ to the denary, we found its factors were 3, 13, 11. The last figure of the numerator shows that it will not divide without remainder by 3. Since $t+8+5+4$, or 27, is not divisible by 11, the number is not; and lastly, since $4+8-(5+t)$ are not divisible by 13, the number is not. Hence we know without finding the G.C.M. that $\frac{t854}{2e9}$ is in its lowest terms.

47. $23\cdot3\dot{5}$ (senary)

$= \frac{2335 - 233}{50}$ (senary), or $\frac{2102}{50}$

$= \frac{470}{30}$ or $\frac{47}{3}$ (denary).

48. 40)3· grs.

3)·03333 scr.

13)·01111 drs.

22)·000342 oz.

Ans. ·000013, etc. lbs.

49. ·100)55 sq. in. (duodenary)

9)·55 ft.

·0728 yds.

4

t1)·24t8 qr. yds.

34)·002t468 . . po.

4)·0000t39 . . ro.

·000026e . . ac.

This can be proved by reducing 55 (duodenary) to 65 (denary), reducing this to acres, which is = ·0000103 . . acres, and this will be found to be = ·000026*e* . . (duodenary) acres. Ans.

50. 1 po. 1 yd. 1 ft. 1 in. $= 20\frac{7}{12}$ ft. (denary), or 18·7 (duodenary).

3 yds. 2 ft. 7 in. $= 11\frac{7}{12}$ ft. (denary) = *e*·7 (duodenary).

5 yds. 2 ft. 11 in. $= 17\frac{11}{12}$ ft. (denary) = 15·*e* (duodenary).

18·7 ft.
e·7 „

1001
16*t*5

17*t*·51 sq. ft.
15·*e*

16267*e*
83415
17*t*51

257*e*·90*e* cub. ft.

2 sq. yds. 8 ft. 87 in. $= 26\frac{87}{144}$ (denary) = 22·73 (duodenary).

22·73′)257*e*·90′*e*(114·69 . .
2273

3089
2273

·*t*160
8*t*50

1310*e*
11376

19550
17*e*53

15*e*9

114
12

13 twelves
12

160 units (denary)
12)9· (twelfth)²s

12)6·75 twelfths

·5625 units.

Ans. 160·56 . . ft. (denary).

CHAPTER X.

1. In the 1 lb. of metal there is $\frac{3}{4}$ lb. copper, in the 2 lbs. of metal there is $\frac{4}{3}$ lb. copper; $\therefore$ in the mixture there are $\frac{3}{4}+\frac{4}{3}$, or $\frac{25}{12}$, or $2\frac{1}{12}$ lbs. copper. Ans.

2. 8 lbs. of tea and 3 lbs. of sugar cost 22s., 5 lbs. of tea and 4 lbs. of sugar cost $15\frac{1}{6}$s., 32 lbs. of tea and 12 lbs. of sugar cost 88s., 15 lbs. of tea and 12 lbs. of sugar cost $45\frac{1}{2}$s.; $\therefore$ 17 lbs. of tea cost $42\frac{1}{2}$s., or 1 lb. of tea costs $\frac{85}{2}\times\frac{1}{17}$s., or 2s. 6d.; $\therefore$ 3 lbs. of sugar cost 2s., or 8d. a lb. Ans.

3. See Hints, p. 330.

4. The sovs. are in value 1 guinea more than the shillings, and the shillings are in value $1\frac{1}{2}$ guineas more than the sixpences; $\therefore$ the sovs. are in value $2\frac{1}{2}$ guineas more than the sixpences. If, then, we take away from £8, 6s. 6d. ($2\frac{1}{2}+1\frac{1}{2}$), or 4 guineas, the remainder can be divided into three equal parts, to the quotient of which if we add $2\frac{1}{2}$ guineas, we get the value of the sovereigns, and if we add $1\frac{1}{2}$ guineas we get the value of the shillings, and the quotient itself will be the value of the sixpences.

$\frac{1}{3}$(£8, 6s. 6d. − £4, 4s. 0d.) = $\frac{1}{3}$ of £4, 2s. 6d., or £1, 7s. 6d., or 55 sixpences.

The shillings are £1, 7s. 6d. + £1, 11s. 6d., or 59 shillings; and the sovereigns £1, 7s. 6d. + £2, 12s. 6d., or 4. Ans.

5. See Hints, p. 330.

8)132288

8)16536

3)2067

13)689

53

Factors are 2.2.2.2.2.2.3.13.53.

Since any number can be expressed by the formulas $3m$, or $3m\pm1$, and no number of the form of $3m$ can be prime, if m were odd, $3m\pm1$ would be even, and hence not prime; therefore in $3m\pm1$, m must be of the form $2n$, or $6n\pm1$. Ans.

6. A ran 50 yards more than B, and this was $\frac{1}{3}$ of the course; $\therefore$ the course was 150 yds. The time of the race is not wanted.

7. $\frac{165}{45045} = \frac{33}{9009} = \frac{3}{819} = \frac{1}{273}$.

$\frac{1065}{38766} = \frac{5 \times 213}{182 \times 213} = \frac{5}{182}$ Ans.

And $\frac{5}{182} - \frac{1}{273} = \frac{15-2}{91.2.3}$

$= \frac{13}{91.2.3} = \frac{1}{7.2.3} = \frac{1}{42}$ Ans.

And $\frac{1}{273} \div \frac{5}{182} = \frac{1}{\cancel{273}_{3}} \times \frac{\overset{2}{\cancel{182}}}{5}$

$= \frac{2}{15}$ Ans.

$$\frac{2\frac{1}{13} \text{ of } \frac{5}{9} - 1\frac{1}{8} \text{ of } \frac{2}{3}}{1\frac{1}{21} \div 5\frac{4}{7}}$$

$$= \frac{\frac{27}{13} \text{ of } \frac{5}{9} - \frac{9}{8} \text{ of } \frac{2}{3}}{\frac{22}{21} \times \frac{7}{39}} = \frac{\frac{15}{13} - \frac{3}{4}}{\frac{22}{3 \times 39}}$$

$$= \frac{60-39}{13 \times 4} \times \frac{3 \times 39}{22} = \frac{189}{88}$$

$= 2\frac{13}{88}$ Ans.

8. (i.)

$$\frac{6642{\cdot}0666}{7358000} = \frac{66420666}{73580000000}$$

$$= \frac{9027}{10000000} = {\cdot}0009027. \text{ Ans.}$$

(ii.) $\frac{{\cdot}066420666}{{\cdot}007358} = \frac{66420666}{7358000}$

$= \frac{9027}{1000} = 9{\cdot}027$. Ans.

(iii.) $\frac{875 \times \dot{2}7\dot{0}}{{\cdot}125 + {\cdot}125\dot{6}75} + \frac{3}{53}$

$$= \frac{\frac{875}{1000} \times \frac{270}{999}}{\frac{250675 - 250}{999000}} + \frac{3}{53}$$

$$= \frac{875 \times 270}{250425} + \frac{3}{53}$$

$$= \frac{\overset{5}{\cancel{35}} \times 270}{\underset{1431}{\cancel{1001}}} + \frac{81}{53 \times 27}$$

$= \frac{1431}{1431} = 1$. Ans.

9. See Hints, p. 330.

10. $\frac{3937079 \times 10000000 \times 4}{12 \times 3 \times 1760}$,

or $\frac{19685395}{792}$ miles.

792)19685395(24855 miles
1584
3845
3168
6773
6336
4379
3960
4195
3960
235
8
1880(2 fur.
1584
296
40
11840

11840(14 po.
792
3920
3168
752
$5\frac{1}{2}$
3760
376
4136(5 yds.
3960
176
3
528(0 ft.
12
6336(8 in.
6336

Ans. 24855 mls. 2 fur. 14 po. 5 yds. 0 ft. 8 in.

11. $\frac{£65}{9\frac{3}{4}\text{d.}} = \frac{65 \times 20 \times 12 \times 4}{39}$

= 1600 francs, which the creditor receives.

$65 \times 25 = 1625$, the francs he is owed;

∴ he loses 25 francs. Ans.

12. A cubic decimetre contains $3{\cdot}937 \times 3{\cdot}937 \times 3{\cdot}937$ cub. in., each of which weighs $252{\cdot}45$ grs.; $\therefore$ a kilogramme weighs

$$\frac{3{\cdot}937 \times 3{\cdot}937 \times 3{\cdot}937 \times 252{\cdot}45}{7000} \text{ lbs.},$$

$$\text{or } \frac{3937 \times 3937 \times 3937 \times 25245}{7000000000000000},$$

$$\text{or } \frac{15499969 \times 3937 \times 3600}{100000000000000},$$

$$\text{or } \frac{155 \times 3937 \times 36}{10000000}, \text{ or } 2{\cdot}196, \text{ etc., lbs.},$$

or 2 lbs. $3\frac{1}{5}$ oz. nearly.

13. See Hints, p. 331.

14. One hectare contains $\frac{35881}{3}$ sq. yds., and these are $= 10000$ sq. metres;

$\therefore$ 1 sq. metre $= \frac{35881}{30000}$ sq. yds.;

$\therefore$ 1 metre

$= \sqrt{1{\cdot}196033 \ldots}$ yds.

$= 1{\cdot}093633 \ldots$ yds.

15. Let us suppose he has to go 120 miles. This would take him 4 hours in the ordinary train and 3 hours in the express, thereby saving an hour, but it costs him $120 \times \frac{1}{2}$d., or 2s. 6d. more; $\therefore$ his time is worth 2s. 6d. an hour.

16. See Hints, p. 331.

17. A cubic metre contains $\frac{394 \times 394 \times 394}{1000}$ cub. in.;

$\therefore$ 1 litre contains $\frac{394 \times 394 \times 394}{1000000} = 61{\cdot}162984$.

18. 1st year he invests $\frac{9}{10}$ of £1000, or £900, for which he receives as additional income $\frac{900}{90} \times 3$, or £30.

2nd year he invests $\frac{9}{10}$ of £1030, or £927, for which he receives as additional income $\text{£}\frac{927}{90} \times 3$, or £30·9.

3rd year he invests $\frac{9}{10}$ of £1060·9, for which he receives an additional income $\text{£}\frac{954{\cdot}81}{90} \times 3$, or £31·827.

4th year he invests $\frac{9}{10}$ of £1092·727, for which he receives an additional income

$$£\frac{983{\cdot}4543}{90}\times 3, \text{ or } £32{\cdot}78181.$$

Therefore his entire income after the 4th purchase will be £1125·50881, or £1125, 10s. $2\frac{143}{1250}$d. Ans.

19. See Hints, p. 331.

20. The express train takes $\frac{1}{32}$ of an hour to travel each mile, the Parliamentary $\frac{1}{14}$ of an hour; ∴ the difference is

$$\frac{1}{14}-\frac{1}{32}, \text{ or } \frac{16-7}{2.7.16}, \text{ or } \frac{9}{2.7.16}$$

of an hour for every mile, but the entire difference is $2\frac{1}{4}$ hrs., and $2\frac{1}{4}$ contains $\frac{9}{2.7.16}\ \frac{9}{4}$

$$\times\frac{2.7.16}{9}, \text{ or } 56 \text{ times, which}$$

is the number of miles. Ans.

21. If the train takes 10 min. to go $4\frac{1}{2}$ miles, it takes $2\frac{2}{9}$ min. to go each mile.

Let us subtract the 10′ he stops and the 30′ he takes to walk to and from the Cambridge station from $3\frac{1}{4}$ hrs., and he does the rest in 2 hrs. *35′, or 155′.* This he does *at the rate of* 15′ a mile one way and $2\frac{2}{9}$′ the other, and $17\frac{3}{9}$ is contained in 155 9 times; hence his railway journey is 9 miles, and the village is 10 miles from his room. Ans.

22. The distance that he walks both ways took him 10 min.; therefore the distance that he walked one way and ran the other took him 15 min.

He walks a mile in $\frac{60}{3\frac{1}{2}}$ min., and runs it in $\frac{60}{7}$ min.; ∴ he takes to walk and run a mile $\frac{180}{7}$ min., and this is contained in 15 min. $\frac{7}{12}$ of a time; therefore the distance that he runs was $\frac{7}{12}$ of a mile.

23. First subtract from £7500 the £500 which the widow has in addition to her other shares.

Each daughter has 1 share,
„ son „ 2 „
∴ 3 daughters have 3 shares,
2 sons „ 4 „
and the wife, „ 7 „
∴ a share is $£\frac{7000}{14}$, or £500,
∴ each daughter has £500,
„ son „ £1000,
and the wife has £7 × 500 + £500, or £4000. Ans.

24. The pursuing army gain $(23-18)$, or 5 miles a day, but they have $(26+3\times18)$, or 80 miles to catch up, and this will take them 16 days to do; hence they come up with the retreating army in $(16+3)$, or 19 days. Ans.

25. We must begin at the end. If 20 is what is left after half, except 8, of what was left after third child had received his division, there must have been $2(20-8)$, or 24 left to divide amongst the last 2. Treating this 24 in the same way, there was $2(24-8)$, or 32 to divide among the last 3. Similarly there was $2(32-8)$, or 48 to divide amongst the last 4, and $2(48-8)$, or 80 amongst them all; and the divisions were 32, 16, 8, 4, 20. Ans.

26. By the principle of proportional parts, explained in Chap. XIX, we could at once find out the times by finding $\frac{3}{3+5}$ of 12 hrs., and $\frac{5}{3+5}$ of 12 hrs., which gives us $4\frac{1}{2}$ hrs. and $7\frac{1}{2}$ hrs.

It can also be done more simply thus:—

If it take 3 units of time to row 1 mile with the stream, it will take 5 *units against* the stream; hence to row over the entire course each way it will take $(3+5)\times30$ units of time, and since this time is 12×60 min., each unit is 3 min., or it will take $3\times30\times3$ min. or $4\frac{1}{2}$ hrs. to row with the stream, and $5\times30\times3$ min. or $7\frac{1}{2}$ hrs. to row against the stream. Ans.

27. If the waistcoat cost 4 units of price, the trousers cost 6 units, and the coat 15 units; $\therefore$ £4, 7s. 6d. contains 25 units of price, or each unit is 3s. 6d.; $\therefore$ the waistcoat cost 14s., the trousers 21s., and the coat 52s. 6d. Ans.

28. This problem, I am sorry to say, cannot be worked by pure arithmetic. Had the 9 gals. been given in the form of $\frac{1}{5}$ of the cask, there would have been no difficulty in doing so.

18 gals. of water are put into the cask, and of this a small quantity is drawn out at the second draw, viz.,

$$\frac{9}{\text{number of gals. in cask}}$$

of 9 gals., or

$$\frac{81}{\text{number of gals. in cask}}\text{ gals.}$$

If then we call this number g, we get $18-\frac{81}{g}=\frac{9}{25}$ of g, or $2-\frac{9}{g}=\frac{g}{25}$.

$g^2 - 50g + 225 = 0$, whence $g = 45$ or 5; $\therefore$ the cask contained 45 gals. Ans.

29. A's time of doing the course : B's time $= 7\frac{1}{2} : 8\frac{1}{3}$, or $9 : 10$; $\therefore$ B's time is $\frac{10}{9}$ of A's time, B's time $+ 140' = 2$ A's time $+ 12$; $\therefore$ $\frac{8}{9}$ A's time $= 128'$, or A took $144'$, and the distance is $\frac{144}{7\frac{1}{2}}$, or 20 miles. Ans.

30. Each egg at 2 a penny cost $\frac{1}{2}$d., each egg at 3 a penny cost $\frac{1}{3}$d.; $\therefore$ 2 eggs cost $\frac{5}{6}$d. But for these 2 eggs he receives $\frac{4}{5}$d.; $\therefore$ he loses $\frac{5}{6} - \frac{4}{5}$, or $\frac{1}{30}$d.; $\therefore$ to lose 1d. he must sell 30 pairs, or 60 eggs. Ans.

31. The £20, which is given to E, is £5 more than $\frac{1}{2}$ the remainder, which is left after A, B, and C have received their shares; $\therefore$ this remainder was $2(20 - 5)$, or £30. Again, this £30 is £10 more than $\frac{1}{3}$ of what was left, after A and B had had their shares; $\therefore$ this remainder was $3(30 - 10)$, or £60. Again, this £60 is £20 more than $\frac{1}{2}$ the remainder, after A has had his share, and this remainder is $2(60 - 20)$, or £80. Lastly, *£80 is £8 more than $\frac{1}{2}$ the money to be divided*; $\therefore$ money to divide is $2(80 - 8)$, or £144. Ans.

32. Let us take as our unit $\frac{1}{2 \cdot 4 \cdot 3 \cdot 3 \cdot 5}$ of the daily wage of either of the men. The first man receives 45 units per hour, the second man receives 40 units per hour, and the third man receives 36 units per hour; or first man receives $45 \times 3 \times (8 + 9)$ units, second man receives $40 \times 3 \times (9 + 10)$ units, third man receives $36 \times 3 (10 + 11)$ units; and these units, viz. 6843, are worth £2, 7s. $6\frac{1}{4}$d. or 2281 farthings, or each unit is $\frac{1}{3}$ farthing, of which first man receives 2295, or $\frac{2295}{3 \cdot 4}$d., or 15s. $11\frac{1}{4}$d.; second man receives 2280 units, or $\frac{2280}{3 \cdot 4}$d., or 15s. 10d.; and third man receives 2268 units, or $\frac{2268}{3 \cdot 4}$d., or 15s. 9d. Ans.

33. What he paid for the 350 was $\frac{2}{2}\frac{5}{8}$ of $\frac{3}{1}\frac{5}{0}\frac{0}{0}$ of 32d., or 100d. We have therefore to divide 350 into two parts, so that one part $\times \frac{1}{8}$d (the price of some of the apples) + the other

part $\times \frac{1}{6}$d. (the price of the rest of the apples) is = 100d.

$$350 \times \tfrac{1}{3} = 116\tfrac{2}{3},$$
$$\text{and } 116\tfrac{2}{3} - 100 = 16\tfrac{2}{3},$$
$$\text{and } \frac{16\frac{2}{3}}{\frac{1}{3} - \frac{1}{5}} = \frac{\frac{50}{3}}{\frac{2}{15}}$$

= 125, the number bought at $\frac{1}{5}$d, and 225 were bought at $\frac{1}{3}$d. Ans.

34. $\frac{£11,\ 12s.}{£435} = \frac{232}{435 \times 20}$

$$= \frac{2}{75} = \frac{\frac{2}{75}}{1} = \frac{\frac{2}{75} \times 100}{100} = \frac{\frac{8}{3}}{100}.$$

Ans.

$$\frac{\frac{8}{3}}{100} \div 3\tfrac{1}{3} = \frac{\frac{8}{3} \times \frac{5}{16}}{100} = \frac{\frac{5}{6}}{100}.$$ Ans.

35. $$\frac{\frac{75}{365} \times 4\frac{1}{3}}{100 + \frac{75}{365} \times 4\frac{1}{3}}$$

of £472, 11s. 9d.

Dividing numerator and denominator by 25,

$$\frac{\frac{3}{365} \times \frac{13}{3}}{4 + \frac{3}{365} \times \frac{13}{3}} \text{ of } 9451\tfrac{3}{4}\text{s.}$$

$$= \frac{13}{1460 + 13} \text{ of } \frac{37807}{4}\text{s.}$$

$$= \frac{1001}{12}\text{s.} = £4,\ 3s.\ 5d.\ \text{Ans.}$$

36. We first have to find what fraction of £472, 11s. 9d. £4, 3s. 5d. is. This is

$$\frac{83\frac{5}{12}\text{s.}}{9451\frac{3}{4}\text{s.}} \text{ or } \frac{1001}{113421}.$$

We secondly have to reduce this fraction to one whose numerator is $\frac{75}{365} \times 4\frac{1}{3}$.

$$\frac{1001}{113421} = \frac{1}{\frac{113421}{1001}}$$

$$= \frac{\frac{75}{365} \times 4\frac{1}{3}}{\frac{113421}{1001} \times \frac{15}{73} \times \frac{13}{3}}$$

$$= \frac{\frac{65}{73}}{\frac{113421}{1001} \times \frac{65}{73}}$$

We lastly have to see by what the denominator of this fraction is greater than $\frac{65}{73}$, or $\frac{75}{365} \times 4\frac{1}{3}$, which we find thus:

$$\left(\frac{113421}{1001} - 1\right)\frac{65}{73} = \frac{112420}{1001} \times \frac{65}{73}$$

$$= \frac{7.11.73.2.10.13.5}{7.11.13.73} = 100.$$

Ans.

37. This is easy.

$$83\tfrac{5}{12}\text{s.} \div \frac{\frac{75}{365} \times 4\frac{1}{3}}{100 + \frac{75}{365} \times 4\frac{1}{3}}$$

$$= \frac{1001}{12} \times \frac{100 + \frac{65}{73}\text{s.}}{\frac{65}{73}} = \frac{1001}{12}$$

$$\times \frac{7365}{65}\text{s.} = \frac{37807}{4}\text{s.} = £472,$$

11s. 9d.

38. See Hints, p. 332.

39. The 41 gals. are sold at 17s. 6d. × 41, and each gal. of spirits is worth $\frac{1}{40}$ of this, or $\frac{41 \times 35}{40 \times 2}$s., or $\frac{287}{16}$s., or 17s. $11\frac{1}{4}$d. Ans.

40.

£	s.	d.
34	12	0
34	7	3
	4	9

4s. 9d. is $\frac{4\frac{3}{4}}{687\frac{1}{4}}$ of £34, 7s. 3d., and $\frac{4\frac{3}{4}}{687\frac{1}{4}} = \frac{19}{2749}$. Ans.

$$\frac{10}{2749} \times \frac{36500}{63} = 4\frac{752}{173187}.$$

41. First we must find what fraction divided by $\frac{63}{36500}$ is $= 4$.

This is $\frac{63 \times 4}{36500} = \frac{63}{9125}$

$\frac{63}{9125}$ is $= \frac{£34,\ 12\text{s.} - x}{x}$.

And we have to find x, which we can do by pure arithmetic, since we perform no operations upon the number we want to find.

$$\frac{63}{9125} = \frac{£34,\ 12\text{s.}}{x} - 1$$

$$\frac{63}{9125} + 1, \text{ or } \frac{9188}{9125} = \frac{£34,\ 12\text{s.}}{x}$$

$$x = £\frac{34\frac{3}{5} \times 9125}{9188}$$

$$£\frac{173 \times 1825}{9188}$$

$$= £34,\ 7\text{s. } 3\tfrac{141}{2297}\text{d.}$$

$\therefore \frac{141}{2297}$d. must be added. Ans.

42. 1 cwt. of wheat measures $\frac{35}{17\frac{1}{4}}$ bus.; $\therefore$ 92 lbs. measure $\frac{92}{112}$ of $\frac{35 \times 4}{69}$ bus. But 1 bus. of oats weighs $\frac{9\frac{3}{4}}{28}$ cwt.; $\therefore$ $\frac{92}{112}$ of $\frac{35 \times 4}{69}$ bus. of oats weigh $\frac{92}{112}$ of $\frac{35 \times 4}{69}$ of $\frac{39 \times 112}{4 \times 28}$ lbs., or 65 lbs.

43. If B be $\frac{1}{3}$ of $\frac{21}{4}$ or $\frac{7}{4}$ of C, and D be $\frac{2}{7}$ of $\frac{35}{4}$ or $\frac{5}{4}$ of C; ∴ C is $\frac{2}{5}$ of D, and B is $\frac{7}{4}$ of $\frac{2}{5}$ or $\frac{7}{10}$ of D.

44. Let us take as our unit the feed of one sheep for one month, then a horse consumes 2 units, and an ox 3 units each month. A's horses consume $3 \times 10 \times 1\frac{1}{2}$ units, A's oxen consume $2 \times 30 \times 2$ units, or altogether A's animals consume $45 + 120 + 325$ units or 490 units, B's horses consume $3 \times 20 \times 1$ units, B's oxen consume $2 \times 40 \times 1\frac{1}{2}$ units, or altogether B's animals consume $60 + 120 + 800$ or 980 units; and since 980 is 2×490, B ought to pay twice as much as A, or $\frac{2}{3}$ of £60, viz., £40, and A ought to pay £20. Ans.

45. If A travels 7 mrs. in 5 hrs. he travels 1 mr. in $\frac{5}{7}$ hr. B travels 5 mrs. in 3 hrs.; ∴ he travels 1 mr. in $\frac{3}{5}$ hr.; ∴ B walks each mr. in $\frac{4}{35}$ hr. less than A does; ∴ as often as 8 hrs. contain $\frac{4}{35}$ hrs. so many mrs. has B to walk before he catches A. $8 \div \frac{4}{35} = 70$; ∴ B walks 70 mrs. before he catches A, and this he does in $70 \div \frac{5}{3}$ or 42 hrs. Ans.

To prove this we can see how far A walks, viz. $(42 + 8)\frac{7}{5}$ or 70 mrs.

46. Let us take as our unit $\frac{1}{2 \cdot 3 \cdot 5}$ of what A or B can do in 6 hrs. or C in 10 hrs. A's and B's work each hour is 5 units, C's work each hour is 3 units. In 10 hrs. A does 50 units, and this being $\frac{1}{3}$ of the work it consists of 150 units. Of the 100 units left to do, B does 14×5 or 70, and C can do the remaining 3×10 in 10 hours. Ans.

47. If we consider his savings in the first year as our unit, in the 2nd, 3rd, etc., he saves $\frac{3}{2}$, $\frac{9}{4}$, $\frac{27}{8}$, $\frac{81}{16}$, $\frac{243}{32}$, and $\frac{729}{64}$, or altogether $\frac{2059}{64}$ units, and £102, 19s. $\div \frac{2059}{64} = \frac{2059}{20} \times \frac{64}{2059}$, or £3, 4s. Ans.

48. $\frac{45\frac{1}{2}}{6\frac{1}{2}}$, or $\frac{91}{13}$, or 7 loaves.

$\frac{45\frac{1}{2}}{7}$, or $\frac{91}{14}$ or $6\frac{1}{2}$;

∴ they must eat $\frac{1}{2}$ a quartern less.

49. Since $\frac{1}{3}$ of the men on whose presence their earnings depend are absent, they all lose $\frac{1}{3}$ of their wage. Ans.

50. Let us take $\frac{1}{3 \cdot 4 \cdot 8\frac{1}{4}}$ of the work as our unit; ∴ the work must consist of 99 units. 3 men can do 99 units in $8\frac{1}{4}$ hours; ∴ each man can do 4 units in 1 hour, and 1 woman can do 3 units in 1 hour. In the second case there are 297 units to do; and each man can do $\frac{6}{5}$ of 4 or $\frac{24}{5}$ units, and each woman $\frac{6}{5}$ of 3 or $\frac{18}{5}$ units. The 7 men do $\frac{7 \times 24}{5}$ units and the 9 women $\frac{9 \times 18}{5}$ units, or together they do $\frac{168 + 162}{5}$, or 66 units each hour, and $\frac{297}{66} = 4\frac{1}{2}$; ∴ they would take $4\frac{1}{2}$ hours. Ans.

Miscellaneous Answers on Part I.

I.

1. 973)368767(379 Ans.
2919
7686
6811
8757
8757
...

2. 57)10003(175
57
430
399
313
285
28

Ans. 175 times, with a remainder of 28.

3. See Hints, p. 332.

4. 15792
393
47376
142128
47376
313
6206569 Ans.

5. $2(30+8) = 2 \times 38 = 76$. Ans.

6. Take away 3 from each, and find the G. C. M. of the remainder.

3567	5547
3	3
9)3564	9)5544
11)396	11)616
4)36	4)56
9	14

G. C. M. $= 9 \times 11 \times 4$ Ans.

7. $5\frac{1}{4}$d., 2s. $7\frac{1}{2}$d., 1s. $7\frac{1}{4}$d., 10s. 1d. Reducing to farthings the numbers are 21, 2×63, 77, 121×4.

L. C. M. $= 3.7.2.3.11.11.2$ farthings $= \frac{7.3.11.11}{4}$ shillings or £31, 15s. 3d.

8. A can give B 16″ start, but A can give C 60″ start; ∴ B can give C 44″ start, but C does $\frac{352}{1760}$ or $\frac{1}{5}$ mile in 44″; ∴ he can do a mile in 220″, or 3′ 40″, and A would take 2′ 40″, and B would take 2′ 56″ Ans.

II.

9.

	1000000
Subt.	1000
	999000
Subt.	$247689\frac{1}{2}$
	$751310\frac{1}{2}$. Ans.

10. The 2nd number is $\frac{2}{3}$ of the 1st, 3rd number is $\frac{3}{4}$ of the 1st, and $\frac{2}{3} \sim \frac{3}{4} = \frac{1}{12}$, but this $\frac{1}{12}$ is 12; ∴ the 1st number is 144. Ans.

11. $\frac{5}{7}$ of number $= 40$; ∴ number 56. Ans.

12. $3\frac{1}{2}$, $4\frac{1}{2}$, $5\frac{1}{4}$, $2\frac{4}{7}$. L. C. M. of 7, 9, 21, 14, is 7.3.3.2 or 126. Ans.

13.

	3791	5232
	17	15
	3774	3)5217
	3)1887	1739
		1258
	629	
	481	481
		37
	4)148	
		111
	37	111

G. C. M. $= 3 \times 37$, or 111. Ans.

14. £379, 2s. 9d. We must take away £23 and £23 − 20 or £3.

£	s.	d.
379	2	9
26	0	0
3)353	2	9
117	14	3

∴ C has £117, 14s. 3d., B has £120, 14s. 3d., and A £140, 14s. 3d. Ans.

15. $\frac{3\frac{1}{2}}{4\frac{1}{4}} = \frac{1}{4\frac{1}{4} \div 3\frac{1}{2}}$

$= \frac{4\frac{1}{4}}{4\frac{1}{4} \times \frac{2}{7} \times 4\frac{1}{4}} = \frac{4\frac{1}{4}}{5\frac{9}{56}}$ Ans.

16.

	s.	d.
$\frac{1}{5}$ of £2, 7s. 9d. =	9	$6\frac{3}{5}$
·75, or $\frac{3}{4}$ of 11s. 6d. =	8	$7\frac{1}{2}$
Difference,	0	$11\frac{1}{10}$

Ans. $11\frac{1}{10}$ pence.

III.

17. $4\frac{1}{2} - \frac{7}{8} = 3\frac{5}{8}$.

$\frac{4}{5}$ of $\frac{10}{8}$ of $\frac{27}{10}$ of $\frac{1}{9} = \frac{4}{5}$.

$3\frac{5}{8} - \frac{4}{5} = 3\frac{25-32}{5\cdot 8} = 2\frac{33}{40}$. Ans.

18. $\frac{5}{4}$ of $\frac{64}{2} = 5 \times 8 = 40$.

Ans.

19. $\frac{3}{5}, \frac{4}{7\frac{1}{2}}, \frac{5}{8\frac{1}{4}}$.

$= \frac{1}{\frac{5}{3}}, \frac{1}{\frac{15}{8}}, \frac{1}{\frac{33}{20}}$

$= \frac{1}{1\frac{2}{3}}, \frac{1}{1\frac{7}{8}}, \frac{1}{1\frac{13}{20}}$.

Since $\frac{7}{8}$ is greater than $\frac{2}{3}$, or $\frac{13}{20}$; $\therefore$ $\frac{4}{7\frac{1}{2}}$ is the least fraction of the three. Ans.

20. $\frac{99}{7}, \frac{143}{9}$

$= \frac{9.11.9}{7.9}, \frac{11.13.7}{9.7}$;

$\therefore$ G. C. M. $= \frac{11}{7 \times 9}$, or $\frac{11}{63}$. Ans.

21. A has $\frac{3}{7}$ of B's, and B has $\frac{4}{11}$ of C's, and C has $\frac{3}{11}$ of D's; $\therefore$ by reciprocals D has $\frac{11}{3}$ of $\frac{11}{4}$ of $\frac{7}{3}$, or $\frac{847}{36}$ of A's. Ans.

22. A has $\frac{36}{847}$ of £21, 3s. 6d. $= \frac{36}{847}$ of $\frac{847}{2}$s. = 18s.

B has $\frac{7}{3}$ of 18s. = 42s.

C has $\frac{3}{11}$ of $\frac{847}{2}$s., or £5, 15s. 6d. Ans.

23. $3(18-2) = 48$. Ans.

24. $\frac{\frac{1}{100}\text{ lbs.}}{\frac{1}{7}\text{ gr.}} = \frac{\frac{12.20.24}{100}}{\frac{1}{7}}$

$= \frac{12.24.7}{5} = 403\frac{1}{5}$. Ans.

IV.

25.

3 sevenths × 4 = 12 twenty-eighths

21 quarters × 7 = 147 twenty-eighths

12 twenty-eighths) 147 twenty-eighths

12 times . . . 3.

Ans. *12 times*, and the remainder is 3 twenty-eighths.

26. Walking, I take 20′ to do a mile. Riding, I take $6\frac{2}{3}$′. ∴ the difference is $13\frac{1}{3}$′ each mile, and this is contained in 20′, $\frac{20 \times 3}{40}$, or $\frac{3}{2}$ time; ∴ the distance is $\frac{3}{2}$ mile, or 1 mile 4 fur. Ans.

27. A figure will make this plain.

Let AB = number required. Near B take C, and let BC = 8. Bisect AC in D. Then AD is the remainder when 29 has been taken away, and AC when 8 has been taken away. But DC = 21; ∴ AB = 2 × 21 + 8, or 50. Ans.

28. We must find the L. C. M. of these sums:

3s. $7\frac{1}{2}$d., 4s. 9d., 5s. $2\frac{1}{2}$d. Reducing to farthings,

$87 \times 2,\ 57 \times 4,\ 249$
$= 2.3.29.19.2.83$ farthings
$= 3.29.19.83$ pence
$= £571,\ 13s.\ 3d.$ Ans.

29.
371324
9
—
34 (nine)^{4}s
9
—
307 (nine)^{3}s
9
—
2766 (nine)^{2}s
9
—
24896 nines
9
—
224068 units (denary).
Ans.

30.

12997	14637
123	12997
69	1640
41	41
287	
287	

Ans. 41.

31. See Hints, p. 333.

32. See Hints, p. 334.

V.

33. $67 \times 2 = 134$.

$$\begin{array}{r} 379 \\ \text{Subtracting } 134 \\ \hline 245 \end{array}$$ Ans.

34. $\dfrac{4 \text{ lbs. } 1 \text{ dr.}}{2 \text{ oz. } 3\frac{1}{3} \text{ dr.}} = \dfrac{1025}{35\frac{1}{3}}$

$$= \frac{1025 \times 3}{106} = \frac{3075}{106}$$

$= 29$ times and 1 over, which 1 is a third of a dram.

35. sq. po. yds.

$$\begin{array}{r} 7 \quad 17 \\ 30\frac{1}{4} \\ \hline 227 \\ 1\frac{3}{4} \\ \hline 228\frac{3}{4} \text{ yds.} \\ 9 \\ \hline 2058\frac{3}{4} \text{ ft.} \\ 144 \\ \hline 8232 \\ 8232 \\ 2058 \\ 108 \\ \hline 296460 \text{ in.} \\ 9 \\ \hline 2668140 \text{ ninths of an inch.} \end{array}$$ Ans.

36. $4\frac{1}{2} - \frac{2}{3} = 4\frac{3-4}{2 \cdot 3} = \frac{23}{6}$, and

$\frac{23}{6} \div 7 = \underline{\frac{23}{42}}$. Ans.

37. $\frac{27}{5} = 5\frac{2}{5}$,

$\frac{35}{7} = 5$;

$\therefore$ 27 does not contain 5 any more times than 35 contains 7, but in the former case there is a remainder of 2 units.

38. $10\frac{3}{5}\text{s.} + £\frac{2}{7} - \frac{4}{11}$ guinea

$= 10\frac{3}{5}\text{s.} + 5\frac{5}{7}\text{s.} - 7\frac{7}{11}\text{s.}$

$= 8\frac{231 + 275 - 245}{5 \cdot 7 \cdot 11}\text{s.}$

$= 8\frac{261}{5 \cdot 7 \cdot 11}\text{s.} = 8\text{s. } 8\frac{52}{385}\text{d.}$

$8\frac{261}{385}\text{s.}$ is $\frac{\frac{3341}{385}}{41}$ of £2, 1s.,

and $\frac{3341}{385} \times \frac{1}{41} = \underline{\frac{3341}{15785}}$. Ans.

39. $49\overline{)1008}\,(20$

$$\begin{array}{r} 98 \\ \hline 28 \end{array}$$

$\therefore$ we have to find what dividend will give a quotient of 60 with a remainder of 14 after it has been divided by 49, which is $60 \times 49 + 14$, or 2954, and 2954 is greater than 1008 by $(2954 - 1008)$, or 1946. Ans.

40. 24 is $\frac{4}{5}$ of the number; ∴ the number is 30. Ans.

VI.

41.
$$317\overline{)100317}\,(316$$
$$951$$
$$521$$
$$317$$
$$2047$$
$$1902$$
$$145$$

$100317 - 145 = \underline{100172}$. Ans.

42. $47 \times 113 + 377 + 17$, or 11720.

43. See Hints, p. 334.

44.
$$\cdot033\overline{)4\cdot700}\,(142$$
$$33$$
$$140$$
$$132$$
$$80$$
$$66$$
$$14$$

Ans. 142 times, and remainder of 14 thousandths.

45. A can run 5×1760, or 8800 yds., whilst B can run $5 \times 1760 - 220$, or 8580;

∴ A's running is $\frac{8800}{8580}$, or $\frac{440}{429}$ of B's.

Similarly, B can run 8800, whilst C can run 8140;

∴ B's running is $\frac{8800}{8140}$ of C.

Again, C's running is $\frac{8800}{8700}$ of B's;

∴ A's running power is $\frac{440}{429}$ of $\frac{440}{407}$ of $\frac{88}{87}$ of D's, and

$$\frac{87.407.429.8800}{88.440.440} = 7846\tfrac{5}{16};$$

∴ whilst A can run 5 miles D can run $7846\frac{5}{16}$ yds., which is $953\frac{11}{16}$ less than 5 miles. Ans.

46. Since A ought to have given him $953\frac{11}{16}$ yds., and only gives him 400 yds., and beats him by a minute, he runs $553\frac{11}{16}$ yds. in a minute, or 1 mile in $\frac{1760 \times 16}{8859}$, or $3\frac{1583}{8859}$ min. Ans.

47. $\frac{3}{13}$ dollar = 1 shilling,
$\frac{4}{6}$ shil. = 1 franc;
∴ 18200 francs
$= \frac{18200 \times 4 \times 3}{5 \times 13}$ dollars
= 3360 dollars. Ans.

48. $\frac{1}{2\frac{1}{3}\frac{1}{4}\frac{1}{5}} = \frac{1}{2\frac{1}{3}\frac{5}{21}} = \frac{1}{2\frac{21}{68}}$
$= \frac{68}{187}$ Ans.

$\frac{1}{2\frac{1}{3}} = \frac{3}{7}$; $\frac{1}{2\frac{1}{3}\frac{1}{4}} = \frac{1}{2\frac{4}{13}} = \frac{13}{30}$.

To compare these fractions see Hints, p. 334.

VII.

49. $\frac{9}{2}$ of 3s. 3d. = $\frac{9}{2}$ of $\frac{13}{4}$s.
= $\frac{117}{8}$, or $14\frac{5}{8}$s., or 14s. $7\frac{1}{2}$d. Ans.

And $\frac{\frac{117}{8}}{23\frac{1}{4}} = \frac{117}{2 \times 93} = \frac{39}{62}$. Ans.

50.

		£	s.	d.
1 qr. at £4, 7s. 6d. =		4	7	6
2 qrs.		8	15	0
11 lbs. 3 oz. $3\frac{1}{8}$ drs. =	of 2 qrs.	1	15	0
		10	10	0 Ans.

51.
$$\begin{array}{r|l} 37)17\cdot00(45 \\ 148 \\ \hline 220 \\ 185 \\ \hline 350 \\ 333 \\ \hline 170 \end{array}$$
Ans. 45 whole times.

52. If $\frac{4}{7} = \cdot\dot{5}71428\dot{5}$,
$\frac{1}{7} = \cdot\dot{1}42857\dot{1}$,
and $\frac{1}{14} = \cdot0\dot{7}14285$ Ans.

53. If we take the cost of a feed as our unit, then $8 \times 3 \times 7 \times 7$ units are $= 2$ guineas, or each unit is worth

$$\frac{2}{8 \times 3 \times 7 \times 7} \text{ guineas.}$$

In the second case it will cost $3 \times 8 \times \frac{7}{2} \times 4 \times 7$, or $3.8.7.2.7$ units, and these are worth $\frac{2.3.8.7.2.7}{8.3.7.7}$, or 4 guineas. Ans.

54. $\frac{\frac{12}{5} \text{ of } £4,\ 7s.\ 9d.}{5}$

$= \frac{12}{5}$ of $£4{\cdot}3875 \div 5$

$£4{\cdot}3875$
$\quad 12$
$25)52{\cdot}6500$
$2{\cdot}106.$ Ans.

55. See Hints, p. 334.

56. $\frac{3}{4\frac{1}{4}}, \frac{5\frac{1}{2}}{6\frac{4}{5}}, \frac{3\frac{1}{2}}{5}$

$= \frac{1}{1\frac{5}{12}}, \frac{1}{\frac{34}{5} \times \frac{2}{11}}, \frac{1}{5 \times \frac{2}{7}}$

$= \frac{1}{1\frac{5}{12}}, \frac{1}{1\frac{13}{55}}, \frac{1}{1\frac{3}{7}}.$

Since $\frac{3}{7}$ is greater than $\frac{5}{12}$ or $\frac{13}{55}$, $\frac{1}{1\frac{1}{3}}$ or $\frac{3\frac{1}{2}}{5}$ is the least.

Reducing them, $\frac{12}{17}, \frac{55}{68}, \frac{7}{10}$

$$\frac{240,\ 275,\ 238}{17.4.5}$$

$\therefore$ the middle one is greater by $\frac{37}{340}$ than the least, and by $\frac{35}{340}$, or $\frac{7}{68}$ than the other. Ans.

VIII.

57. $\frac{6 \text{ lbs. } 11 \text{ oz. } 8\frac{8}{25} \text{ drs.}}{28 \text{ lbs.}}$

$= \frac{43008}{28.16.16.25} = \frac{6}{25}$ Ans.

58.

					£	s.	d.
1 cwt. cost					3	6	8
1 qr. ,,						16	8
lbs.	oz.	drs.					
5	9	$9\frac{3}{5}$	$\frac{1}{5}$		3	4	
1	1	$14\frac{18}{25}$	$\frac{1}{5}$			8	
6	11	$8\frac{8}{25}$			4	0	
3 qrs. 21 lbs. 4 oz. $7\frac{17}{25}$ drs.					3	2	8 Ans.

59. $\frac{3 \text{ drs. avoir.}}{\frac{11}{14} \text{ grs.}} = \frac{\frac{3 \times 7000}{16 \times 16}}{\frac{11}{14}}$

$\frac{3 \times 7000}{16 \times 16} \div \frac{11}{14}$,

or $\frac{3 \times 875}{16} \div \frac{11}{7}$

$\frac{3 \times 875 \times 7}{16 \times 7} \div \frac{11 \times 16}{16 \times 7}$

$176)18375(104$
176

775
704

71

Ans. 104 times and 71 sevenths of a sixteenth of a grain over.

60. 3·14 (quinary)
21·3 »

2002
314
1133

123·442 (quinary)

$·442 = \frac{4}{5} + \frac{4}{25} + \frac{2}{125}$ Ans.

61. $2·3)32·312(11$
23

33
23

1012

Ans. 11 times and $\frac{1012}{1000}$ all expressed in the quaternary, or 5 times and $\frac{70}{64}$ (denary) over.

62. Beginning at the end, $12\frac{1}{4}$ gals. is 1 more than $\frac{3}{4}$ of the second remainder; ∴ the second remainder is $\frac{4}{3}$ of $11\frac{1}{4}$, or 15 gals. Secondly, 15 is 1 more than $\frac{2}{3}$ of first remainder; ∴ the first remainder is $\frac{3}{2}$ of 14, or 21 gals., and lastly, 21 is 1 more than $\frac{1}{2}$ the cask; ∴ the cask contains 2×20, or 40 gals. Ans.

63. $\frac{4\text{s. } 7\frac{1}{2}\text{d.}}{6\text{s. } 11\frac{1}{4}\text{d.}} = \frac{111 \times 2}{333} = \frac{2}{3}$

$= \frac{1}{\frac{3}{2}} = \frac{11 \text{ gals.}}{\frac{3}{2} \times 11 \text{ gals.}}$, or $\frac{11 \text{ gals.}}{16\frac{1}{2} \text{ gals.}}$ Ans.

64. $\frac{1}{118}$ of the fraction is $= \frac{3}{8}$; ∴ the fraction is $\frac{3}{8} \times 118 = \frac{177}{4}$. Ans.

IX.

65. $\cdot 0037')1\cdot 1100'(300$

$1\ 11$

$\cdots$

$$\frac{11\cdot 1}{\cdot 037} = \frac{11100}{37} = 300$$

$$\frac{1\cdot 11}{37000} = \frac{111}{3700000} = \cdot 00003$$

$$\frac{11100}{3\cdot 7} = \frac{111000}{37} = 3000.$$ Ans.

66. $\cdot\dot{5}71428\dot{}$ of a guinea $= \frac{5}{9}\frac{7}{9}\frac{1}{9}\frac{4}{9}\frac{2}{9}\frac{8}{9}$ of $\frac{21}{1}$s. $= \frac{4}{7}$ of $\frac{21}{1}$ = 12s., and £69 less 12s. = £68, 8s.

20)8s.

5)68·4

13·68 Ans.

67. $(37 + 43) \times (37 + 53) \times (43 + 53)$
$= 80 \times 90 \times 96$
$= 7200 \times (100 - 4) = 720000 - 28800$
$= 691200$. Ans.

68. If from a number I subtract 20 more than its quarter and have 100 left, this number must be $\frac{4}{3}$ of (100 + 20) or 160. *But this 160 is* obtained by multiplying a number by $\frac{3}{5}$, hence this number is $\frac{5}{2}$ of 160 or 400, and this 400 is obtained by subtracting something from 500, which must be 100. Ans.

69. See Hints, p. 335.

70. A would have taken $\frac{10}{4\frac{1}{2}}$ hours to have walked the last 10 miles at his original pace, but he walks the last 12 miles in $\frac{12}{4\frac{1}{2} + 1\frac{1}{2}}$, or 2 hours.

$$\frac{10}{4\frac{1}{2}} - 2 = \frac{1}{4\frac{1}{2}} = \frac{2}{9} \text{ hrs.};$$

∴ B drives 2 miles in $\frac{2}{9}$ of an hour, or at the rate of 9 miles an hour. Ans.

71. Whilst A is walking 12 miles, B, who drives half as fast again, is going 18 miles; hence the distance is 30 miles. Ans.

72. Let us take as our unit $\frac{1}{3}$ of each man's daily rations. Then originally there were $3 \times 2000 \times 65$ units in the garrison; after 10 days there were $3 \times 2000 \times 55$. A woman eats $\frac{3}{4}$ as much as a man, and a child eats as much as $\frac{3}{7}$ of a man. Each day after the

arrival of the women and children the men consume 2×2000 units, the women consume $2 \times \frac{3}{4} \times 1000$ units, and the children consume $2 \times \frac{3}{7} \times 1000$ units, or $6357\frac{1}{7}$ units, and these are contained in $3 \times 2000 \times 55$ units

$$\frac{3 \times 2000 \times 55 \times 7}{44500},$$

or $51\frac{8}{89}$ times, which gives the number of days. Ans.

X.

73. 2.3.4.11, 12 (quinary). L. C. M. = 2.3.2.12 = 314 (quinary). Ans.

74. 1·3)43·2(30·12 (senary) Ans.

43

20
13

30
30

..

1·5)27·3̇3̇(18·2̇

15

123
120

33
30

33

denary. senary.

And 18·2 = 30·12

6)18 ·2222

3 sixes 0 units 6

1·33 sixths

6

1·99(sixth)²s

and 1·99 . . . = 2.

75.

		£	s.	d.
375⅔ art. @ £1	=	375	13	4
2		751	6	8
3s. 4d. = ⅙ of £1		62	12	2⅔
5d. = ⅛ of 3s. 4d.		7	16	6⅓
½d. = 1/10 of 5d.		0	15	7⅚
3s. 9½d.		71	4	4⅚
Ans. £1, 16s. 2½d.		£680	2	3⅙

76. His original income was £204, and his new income £(204+136), or £340, of which he receives in money £$\frac{102\times4\times3}{4}$, or £306; ∴ £34 was paid in potatoes, and there were $\frac{102\times4}{3}$ lots or 4×4 bushels of potatoes, and these are worth £34 and $\frac{34\times4}{4\times20}$s. = 5s. Ans.

77.
$$\begin{array}{r} 4179 \\ \underline{6231} \end{array} \qquad \begin{array}{l} 7)10000 \\ \hline 0142 \\ \underline{1326} \end{array}$$

∴ the remainder after dividing by 7 will be 9+21+2+24, or 2+2+3 or 0. 4179 = some threes + 0; ∴ the remainder will be 0, 3, 6, or 9 and some fours + 3; ∴ the remainder will be 3, 7, or 11; ∴ the remainder after dividing by 12 must be 3. Ans.

78. 8 men and 11 boys = 9 men and 8 boys, 3 boys = 1 man's time. Ans.

79. $\frac{1}{6}-\frac{1}{7}=\frac{1}{42}$, and if $\frac{1}{42}$ of a number = 7 the number is 294.

80. $3\cdot72 = 3\cdot642782\ldots$ (nonary) Ans.

$$\begin{array}{r} 9 \\ \hline 6\cdot48 \\ 9 \\ \hline 4\cdot32 \\ 9 \\ \hline 2\cdot88 \\ 9 \\ \hline 7\cdot92 \\ 9 \\ \hline 8\cdot28 \\ 9 \\ \hline \underline{2\cdot52} \end{array}$$

XL.

81. $\frac{3731}{5}$ = some fives and 1; ∴ the remainder is 1, 6, 11, etc. $\frac{3731}{4}$ = some fours and 3; ∴ the remainder after dividing by 4 is 3 or 7 or 11, etc., and the remainder after dividing by 3 is 2 or 5 or 8, 11, etc.; ∴ the remainder after dividing by $3\times4\times5$ is 11. Ans.

82. A sells a horse for $\frac{5}{6}$ of what he gave for it, B sells it for $\frac{8}{7}$ of what he gave for it, and £400 is $\frac{8}{7}$ of what C gave for it. Since £400 is $\frac{8}{7}$ of C's buying price, B sold it for $\frac{7}{8}$ of £400 or £350, £350 is $\frac{8}{7}$ of B's buying price;

$\therefore$ A sold it for $\frac{7}{8}$ of £350, or $\frac{7 \times 175}{4}$, and $\frac{7 \times 175}{4}$ is $\frac{5}{6}$ of what A gave for it, or A gave $\frac{6}{5}$ of $\frac{7 \times 175}{4}$, or £$\frac{735}{2}$.

Ans. £367, 10s.

83. If $\frac{4}{5}$ of $\frac{15}{17}$ of $\frac{3}{8}$ = £$\frac{3000}{175}$, the whole ship is worth

$$\frac{5 \cdot 17 \cdot 8 \cdot 3000}{4 \cdot 15 \cdot 3 \cdot 175},$$

and $\frac{2}{3}$ of $\frac{3}{7}$ of it is worth

$$£\frac{2 \cdot 3 \cdot 5 \cdot 17 \cdot 8 \cdot 3000}{3 \cdot 7 \cdot 4 \cdot 15 \cdot 3 \cdot 175},$$

or £18, 10s. $0\frac{40}{41}$d. Ans.

84. See Hints, p. 336.

85.

47123	
11	
7	
121	
518353	11 times
329861	70 times
57001883	12100 times
574005263	Ans.

86. $\frac{37}{2} - \frac{55}{3} = \frac{111 - 110}{2 \cdot 3} = \frac{1}{6}$.

$\frac{56}{3} - \frac{37}{2} = \frac{112 - 111}{3 \cdot 2} = \frac{1}{6}$.

Ans.

87. If their halves differ by 3, the numbers themselves must differ by 6. Again, if their thirds differ by 2, they must differ by 6, and the smallest numbers that differ by 6 which will divide by 2 and 3 are 6 and 12. Ans.

88. Let us take as our unit a younger daughter's share. The widow receives $\frac{1}{2}$ of (20000 − 1000) + 1000, or 10500. From the remainder, 9500, we must first take away for eldest daughter £500, for 2 younger sons, £1000, and for eldest son, £1500, which leaves £9500 − 3000, and this £6500 ÷ 8 gives us a younger daughter's share, viz. £812, 10s.; the eldest daughter and younger sons, £1312, 10s.; the eldest son, £2312, 10s.; and, as before, the widow, £10,500.

XII.

89. £$\frac{7}{107}$ of $\frac{100}{1} \sim$ £$\frac{7}{100}$ of $\frac{100}{1}$ = £$\frac{700 \sim 749}{107}$

= £$\frac{49}{107}$,

and this multiplied and divided by 100

= $\frac{7}{107}$ of $\frac{7}{100}$ of £100. Ans.

90. 317 is greater than $\frac{2}{3}$ of a number by 11; ∴ the number is $\frac{2}{3}(317-11)$, or $\frac{3}{2}$ of 306 $=459$. Ans.

91. After the leakage there were $\frac{10}{11}$ of $\frac{6}{7}$ of the cask of brandy left, and this was equal to 29 gallons; ∴ the cask originally held $\frac{7}{6}$ of $\frac{11}{10}$ of 29, or $37\frac{1}{60}$ gals. Ans.

92.

712	802	1
710	712	
2	80	
	8	

G. C. M. 2.

93. See Hints, p. 336.

94. 462. Since $4+6+2$ = number divisible by 6, and $6=7-1$; ∴ 462 in the septenary scale is divisible by six; or thus—

$$2=2,$$
$$6\times 7=(6\times 6)+6,$$
$$4\times 49=4\times 48+4.$$

Now, 6×6 and 4×48 will divide by 6, and since $2+4+6$ will also divide by 6, the number will.

Again, since $4+2=6$, or $4+2-6=0$, the number will divide by 8, or $(7+1)$. Reason as for elevens (radix + 1) in the denary scale.

95.

$$5)605$$
$$121$$

And since $121=11\times 11$,

$$605=5\times 11\times 11,$$
$$5\times 11=55,$$
$$\frac{605}{5\times 11}=11;$$

∴ 55 is the same measure of 605 that it is the multiple of 5. Ans.

96. What A wins B loses; ∴ the 10s. changing hands makes a difference of £1 between their money. But the difference between $\frac{1}{3}$ and $\frac{2}{5}$ is $\frac{2}{15}$, and if $\frac{2}{5}$ of A's money is = £1, he now has 30s., or originally they both had £1. Ans.

PART II.

CHAPTER XI.

1 to **5**. See Hints, p. 337.

6. Since we have the two means, we must multiply them together, and divide by the remaining known term; thus:—
$2\frac{1}{2} \times 3\frac{1}{4} \div 2\frac{1}{7} = \frac{5}{2} \times \frac{13}{4} \times \frac{7}{15}$
$= \frac{91}{24}$, or $3\frac{19}{24}$. Ans.

7. Here we have the extremes given us:—
$3\frac{1}{3} \times 5\frac{1}{3} \div 4 = \frac{10}{3} \times \frac{16}{3} \times \frac{1}{4} = \frac{40}{9}$
$= 4\frac{4}{9}$. Ans.

8. $4\frac{1}{4} \times \frac{3}{17} \div 3\frac{1}{3} = \frac{17}{4} \times \frac{3}{17}$
$\times \frac{3}{10} = \frac{9}{40}$. Ans.

9. $8 \times 3\frac{1}{7} \div 5\frac{1}{2} = \frac{8}{1} \times \frac{22}{7} \times \frac{2}{11}$
$= \frac{32}{7} = 4\frac{4}{7}$. Ans.

10. See Hints, p. 337.

11. $x : 6 = 7 : 8$. $6 \times 7 \div 8$
$= \frac{21}{4} = 5\frac{1}{4}$. Ans.

12. See Hints, p. 338.

13. $7 : x = 4 : 5$. $7 \times 5 \div 4$
$= 8\frac{3}{4}$. Ans.

14. $7 \times 5 = 6 : x$. $6 \times 5 \div 7$
$= 4\frac{2}{7}$. Ans.

15. $3 : x = 7\text{s.} : 4\text{s. } 1\text{d.}$
$3 \times 49\text{d.} \div 84\text{d.} = 1\frac{3}{4}$. Ans.

16. $x : 6 = 3$ (2 lbs. : $3\frac{7}{16}$ lbs.). $3 \times 6 \times 32$ oz. $\div 55$ oz., or $10\frac{26}{55}$. Ans.

17, 18. See Hints, p. 338.

19. $x : 2\frac{1}{2} = \frac{3}{5} : \frac{1}{7}$. $2\frac{1}{2} \times \frac{3}{5} \div \frac{1}{7}$
$= \frac{5}{2} \times \frac{3}{5} \times \frac{7}{1} = \frac{21}{2}$, or $10\frac{1}{2}$. Ans.

20. $\frac{3{\cdot}4 \times 5{\cdot}6}{1{\cdot}2} = 15{\cdot}8\dot{6}$. Ans.

21. $\frac{1{\cdot}2 \times 3{\cdot}4}{5{\cdot}6} = {\cdot}7\dot{2}85714\dot{2}$.

22. $\frac{1{\cdot}2 \times 5{\cdot}6}{3{\cdot}4}$
$= 1{\cdot}9764705882352941\dot{1}$.

23. $\frac{2\cdot\dot{3}\times 3\cdot\dot{4}}{1\cdot\dot{1}} = \frac{\frac{21}{9}\times\frac{31}{9}}{\frac{10}{9}}$

$= \frac{7\times 31}{3\times 10} = \underline{7\cdot 2\dot{3}}$. Ans.

24. $\frac{1\cdot\dot{2}}{2\frac{1}{2}} \times \frac{17\frac{1}{2}}{5} \times \frac{15}{2\cdot\dot{4}} = \frac{\frac{11}{9}}{\frac{5}{2}}$

$\times \frac{\frac{35}{2}}{\frac{5}{1}} \times \frac{\frac{15}{1}}{\frac{22}{9}} = \frac{11\times 2}{5\times 9} \times \frac{7}{2} \times \frac{15\times 9}{22}$

$= \frac{21}{2} = \underline{10\cdot 5 \text{ or } 10\frac{1}{2}}$. Ans.

25. 3s. : 10s. $= \frac{3}{10}$

£	s.	d.
	3	6 8
		3
10)	10	0 0
	£1	0 0 Ans.

26.

oz.	drs.	
4	2	(Avoir.)
	3	
11)	12	6
	1	2 Ans.

27.

cwt.	qr.	
5	1	$\div \frac{7}{8}$
	3	
15	3	
2	1	Ans.

28. $\frac{1}{3} : \frac{1}{13} = \frac{13}{3}$

£	s.	d.	
2	3	4	$\div \frac{13}{3}$
		3	
13)	6	10	0
	10	0	Ans.

29. $\frac{\cdot 03}{\cdot 005} \sim \frac{\cdot 0\dot{3}}{\cdot 00\dot{5}} = 6 \sim \frac{\frac{3}{990}}{\frac{5}{990}}$

$= 6 \sim 6\frac{3}{5} = \frac{3}{5}$, or $\underline{\cdot 6}$. Ans.

30. $\frac{2}{7} \sim \frac{4}{9} = \frac{18 \sim 28}{63}$

$= \underline{\frac{10}{63}}$. Ans.

31. $\frac{4}{7} + \frac{3}{5} = \frac{20\times 21}{35} = \frac{41}{35}$;

∴ ratio required is

41 men : 35 men. Ans.

32. $\frac{4}{11} - \frac{2}{7} = \frac{28-22}{77} = \frac{6}{77}$

∴ ratio required is

6 lbs. : 77 lbs. Ans.

33. $\frac{7}{8}$ of $\frac{4\times 16}{182} = \frac{32}{65}$

∴ ratio required is 32s. : 65s. Ans.

34. $\frac{3}{7} = \frac{3}{3+4} = \frac{\frac{3}{2}}{\frac{3}{2}+2}$

$= \underline{\frac{1\frac{1}{2}}{3\frac{1}{2}}}$. Ans.

35. $\frac{10}{9} = \frac{9+1}{9} = \frac{27+3}{27}$

$= \frac{30}{27}$. Ans.

36. $\frac{£\ 2,\ 10s.}{£44,\ 3s.\ 4d.} = \frac{2\frac{1}{2}}{44\frac{1}{6}}$

$= \frac{3}{53} = \frac{3}{3+50} = \frac{6}{6+100}$

$= \frac{6}{106}$. Ans.

37. $\frac{£41,\ 13s.\ 4d.}{£44,\ 3s.\ 4d.} = \frac{41\frac{2}{3}}{44\frac{1}{6}}$

$= \frac{50}{53} = \frac{50}{50+3} = \frac{100}{100+6}$

$= \frac{100}{106}$. Ans.

38. $\frac{2}{3} = \frac{2}{2+1} = \frac{\frac{2}{3}}{\frac{2}{3}+\frac{1}{3}} = \frac{\frac{2}{3}}{1}$. Ans.

39. $\frac{2}{2} = \frac{2+1}{2} = \frac{20+10}{20} = \frac{30}{20}$.

Ans.

40. $\frac{2}{3\frac{2}{5\frac{1}{2}}} \times \frac{3\frac{1}{4}}{\frac{2}{37}} \times \frac{2\frac{1}{1\frac{1}{2}}}{5\frac{1}{2}}$

$= \frac{2}{3\frac{4}{11}} \times \frac{13 \times 37}{2 \times 4} \times \frac{2\frac{2}{3}}{1\frac{1}{2}}$

$= \frac{2 \times 11}{37} \times \frac{13 \times 37}{2 \times 4} \times \frac{8 \times 2}{3 \times 11}$

$= \frac{52}{3} = 17\frac{1}{3}$. Ans.

CHAPTER XII.

1. I. 1s. $10\frac{1}{2}$d. : 5s. $1\frac{1}{2}$d. = 78 lbs. : x lbs.

$5\frac{1\frac{1}{2}}{12} \times 78 \div 1\frac{10\frac{1}{2}}{12}$ lbs.,

or $5\frac{1}{8} \times 78 \div 1\frac{7}{8}$, or $\frac{41 \times 78 \times 8}{8 \times 15}$,

or $213\frac{1}{5}$ lbs. of tea. Ans.

2. Unitary Method.

If 1000 sov. weigh $257\frac{4}{5}$ oz.,

1 sov. weighs $\frac{1289}{5 \times 1000}$ oz.,

192 sov. weigh $\frac{1289 \times 192}{5 \times 1000}$ oz.,

or 4 lbs. 1 oz. 9 dwt. $22\frac{106}{125}$ grs. Ans.

3. Fractional Method.

The answer being less than the given qrs., etc., the fraction must be a proper one.

152 sacks

$\times \frac{18 \text{ qrs. } 7 \text{ bush. } 1 \text{ pk.}}{65 \text{ qrs. } 2 \text{ bush. } 2 \text{ pks.}}$,

or $152 \times \frac{151\frac{1}{4}}{522\frac{1}{2}}$ sacks,

or $152 \times \frac{605}{1045 \times 2}$ sacks,

or 44 sacks. Ans.

4. D. 2 ro. 15 po. : 8 ac. 17 po. $= £59\frac{3}{8} : x$.

$\frac{1297 \times 475}{95 \times 8}$, or £810, 12s. 6d. Ans.

5. D. 17 yds. : 120 yds. $= 88\frac{6\frac{1}{2}}{12}$s. $: x$.

$£\frac{120 \times 2125}{17 \times 2 \times 12 \times 20}$,

or £31, 5s. Ans.

6. I. $18\frac{1}{4} : 13\frac{8\frac{1}{4}}{12}$

$= 24$ E. e $: x$.

$13\frac{11}{4 \times 4} \times 24 \div \frac{73}{4}$ yds.,

or $\frac{219 \times 24 \times 4}{4 \times 4 \times 73}$, or 18 yds. Ans.

7. Let us take as our unit a man's day's work; then there are 8×27 units to be done, and 12 men would do these in $\frac{8 \times 27}{12}$ days, or 18 days. Ans.

8. D. 4 cwts. : 1 lb. $= 16$ guineas $: x$.

$\frac{1 \times 16 \times 21}{4 \times 4 \times 28}$s., or $\frac{3}{4}$s., or 9d. Ans.

9. If £4, 1s. 6d. will buy 12 bush., 3d. will buy $\frac{12}{326}$ bush., and 4401 threepences will buy $\frac{12 \times 4401}{326}$ bush., or *162 bush. Ans.*

10. I. 10 men : 3 men $= 6\frac{2}{3}$ days $: x$.

$\frac{3 \times 20}{3 \times 10} =$ 2 days. Ans.

11. D. £29, 18s., 3d. : £9, 19s. 5d. $= 45$ yds. $: x$.

$\frac{(2400 - 7) \times 45}{(240 \times 30) - 21}$, or $\frac{2393 \times 45}{7179}$, or 15 yds. Ans.

12. D. 50 mls. : 120 mls. $= 12\frac{1}{2}$s. $: x$.

$\frac{120 \times 25}{2 \times 50}$, or £1, 10s. Ans.

13. Using Fractional Method,

$17 \times \frac{£5, 12s., 6d.}{£6, 7s., 6d.}$, or $17 \times \frac{5\frac{5}{8}}{6\frac{3}{8}}$

$17 \times \frac{45}{51}$, or 15 men. Ans.

14. Each horse costs $\frac{£11, 6s., 0\frac{1}{2}d.}{25}$ per week;

$\therefore$ 13 horses will cost $\frac{13 \times 5425}{2 \times 25}$,

or $\frac{13 \times 217}{2}$d. for a week;

and $\therefore$ $\frac{13 \times 31}{2}$d. for a day;

and $\therefore$ $\frac{13 \times 31 \times 60}{2}$d. for the 60 days from September 1 to October 31, and this $=$ £50, 7s., 6d. Ans.

The answer given in the earlier editions is calculated for 61 days.

15. If we take as our unit a ıan's work for a day, there ·e 171 × 12 units to be done, ıd this, $\frac{171 \times 12}{19}$ or 108 men ›uld do in 19 days. Ans.

16. Let us take as our unit ıe cost of carrying 1 ton 1 ile; then there are 36 × 144 ıits of price to use; hence $\frac{5 \times 144}{48}$ is the number of miles ıat 48 tons could be carried, : 108 miles. Ans.

17. Between 10 hrs. 15 min. M., and 5 hrs. 45 min. P.M., ıere are $7\frac{1}{2}$ hrs.; hence the dis- ıce travelled is $7\frac{1}{2} \times 7\frac{1}{2}$, or $5\frac{1}{4}$ miles. Ans.

18. D. £$1063\frac{3}{4}$: £518 = £1150 : x.

$$\frac{518 \times 1150 \times 4}{4255},$$

r $$\frac{2 \times 7 \times 37 \times 5 \times 23 \times 10 \times 4}{5 \times 23 \times 37},$$

r £560. Ans.

19. Taking the same unit s in 15, there are 3 × 60 units o be done, which would be lone in 20 days by $\frac{3 \times 60}{20}$, or) men. Ans.

20. 1 man's wages are £$\frac{28\frac{4}{5}}{36}$;

∴ 60 men's wages are $\frac{144 \times 60}{36 \times 5}$, or £48. Ans.

21. D. 1d. : £1000000 = £1 : x.

£$\frac{1000000 \times 20 \times 12 \times 1}{1}$,

or £240000000. Ans.

22. To use the Fractional Method,

£$4736\frac{1}{4} \times \frac{2\frac{2}{3}}{20}$, or £$\frac{18945 \times 8}{4 \times 3 \times 20}$,

or £631, 10s. Ans.

23. With the stream he can row each hour $3 + 2\frac{1}{2}$ (of which the stream deprived him) $+ 2\frac{1}{2}$ (with which the stream helps him), or 8 miles; ∴ in $3\frac{1}{2}$ hrs. he can row 28 miles. Ans.

24. By Fractional Method,

£$416\frac{2}{3} \times \frac{1}{\frac{1}{6}}$, or £$\frac{1250 \times 6}{3}$,

or £2500. Ans.

25. By Fractional Method,

£$4725 \times \frac{12}{30}\frac{37}{00}$,

or £3050, 15s., 6d. Ans.

26. 21s. × $\frac{11 \text{ pts.}}{18 \text{ gals.}}$,

or $\frac{21 \times 11}{18 \times 4 \times 2}$s., or $\frac{77}{4}$d.,

or 1s. $7\frac{1}{4}$d. Ans.

27. £15, 10s. 6d. $\times \frac{11 \text{ ft.}}{18 \text{ yds.}}$

$= 310\frac{1}{2}\text{s.} \times \frac{11}{18 \times 3}$,

or $\frac{621 \times 11}{2 \times 18 \times 3}$s.,

or £3, 3s. 3d. Ans.

28. D. 5″ : 1 hr. = 11 yds : x.

$\frac{60 \times 60 \times 11}{5 \times 1760}$, or $4\frac{1}{2}$ miles. Ans.

29. D. $17\frac{1}{2}$ mls. : 14 mls. $= 15' : x$.

$\frac{14 \times 15 \times 2}{35}$ min., or 12′. Ans.

30. D. 28 qrs. : 100 qrs. $= 5\frac{1}{4}$ ac. : x.

$\frac{100 \times 21}{28 \times 4}$ ac., or 18 ac. 3 ro. Ans.

31. There are 57×9 units (see **15**) of work to be done. and this could be done in 27 days by $\frac{57 \times 9}{27}$, or 19 men. Ans.

32. $£5\frac{1}{2} \times \frac{9}{11}$ is the price of 9 yds., and £135 contains this price, $\frac{135 \times 2 \times 11}{11 \times 9}$, or 30 *times, which gives us the number of pieces.* Ans.

33. I lose on 8s. 4d. $\frac{5\text{s.}}{£2}$, or $\frac{1}{8}$ of 8s. 4d., or 1s. $0\frac{1}{2}$d. Ans.

34. I gain on 8s. $\frac{£5}{£3}$, or $\frac{5}{3}$ of 8s., or 13s. 4d. Ans.

35. D. 3′ 15″ : 1 hr. = 2 mls. 4 fur. 20 po. : x.

$\frac{60 \times 20\frac{1}{2}}{3\frac{1}{4} \times 8}$ miles, or $\frac{60 \times 41 \times 4}{2 \times 13 \times 8}$,

or $47\frac{4}{13}$ miles. Ans.

36. $£4\frac{1}{2} \times \frac{750}{100}$, or $£\frac{9 \times 15}{2 \times 2}$,

or £33, 15s. Ans.

37. 1 year $\times \frac{100}{80}$,

or 1 yr. 3 mo. Ans.

Note.—The £100 being involved in either case, is not used in the solution.

38. In 2 hrs. 15 min. A walks $2\frac{1}{4}$ of 4 miles or 9 miles. And B would take $\frac{9}{15}$ hrs., or 36 min. to do this. Ans.

39. When A catches B he has walked $\frac{12}{3\frac{1}{2}}$ or $\frac{12 \times 2}{7}$ hrs.; but A has only been on the road $\frac{12 \times 2}{7} - 2\frac{1}{4}$ hrs., and in this time he does 12 miles;

his rate is $12 \times \frac{1}{\frac{12 \times 2}{7} - 2\frac{1}{4}}$,

$12 \times \frac{28}{96 - 63}$ miles,

r $10\frac{3}{11}$ miles an hour. Ans.

40. D. $5\frac{13}{20}$ oz. : $28\frac{1}{4}$ lbs. 22 sov. : x.

$\frac{113 \times 12 \times 22 \times 20}{4 \times 113}$ sov.,

or 1320 sov. Ans.

41.

buy buy sell sell

$\frac{11\frac{1}{2}}{12}$d. : 100d. $= \frac{115}{100}$d. : x.

$\frac{100 \times 115 \times 2 \times 12}{100 \times 23}$d., or 10s.

Ans.

buy buy sell sell

D. 100d. : $\frac{9}{10}$d. $= 104\frac{1}{6}$d. : x

lling price of each egg is

$\frac{9 \times 625}{100 \times 6 \times 100}$d. or $\frac{15}{16}$d.; and ∴

r 1 doz. I shall receive

$\frac{15 \times 12}{16}$, or $11\frac{1}{4}$d. Ans.

sell sell cost cost

D. $91\frac{7}{8}$d. : 35d. = 100d. : x;

his cost of what he sold

r 35d. is $\frac{35 \times 100 \times 8}{735}$d., or

$3\frac{2}{21}$, and ∴ he lost $3\frac{2}{21}$d.

Ans.

44. I. 14s. $10\frac{1}{2}$d. : 15s. 7d. $= 31\frac{1}{2}$ gals. : x.

$\frac{187 \times 63 \times 8}{2 \times 119 \times 12}$, or 33 gals. Ans.

45. I. 14s. $10\frac{1}{2}$d. : 15s. 7d. $= 31\frac{1}{2}$ gals. : x, which gives us, as in **44**, 33 gals.; ∴ $1\frac{1}{2}$ gals. of water must be added.

46. Each creditor receives $\frac{790}{1680}$,

or 9s. $4\frac{6}{7}$d. for every £1. Ans.

47. D. 50s. : 60s. = 6d. : x.

$\frac{60 \times 6}{50}$d. $= 7\frac{1}{5}$d. Ans.

48. I. 60s. : 50s. $= 3\frac{1}{2}$ lbs. : x.

$\frac{50 \times 7}{60 \times 2}$ lbs., or 2 lbs. $14\frac{2}{3}$ oz.

Ans.

49. I paid $35 \times 17\frac{1}{2}$s., but of this I received $\frac{96}{100}$, or $\frac{24}{25}$ of $\frac{35}{1} \times \frac{35}{2}$s., but I sold 35 + 14 gals.; ∴ the price I received for each gal. was $\frac{24}{25}$ of $\frac{35}{1} \times \frac{35}{2} \times \frac{1}{49}$,

or 12s. a gal. Ans.

50. Buying price of what is sold for £$25\frac{13}{20}$ is $\frac{100}{108}$ of $25\frac{13}{20}$, and £24 is $\frac{24}{\frac{100}{108} \text{ of } \frac{513}{20}}$ of this buying price,

or $\frac{24 \times 108 \times 20}{100 \times 513}$,

or $\frac{24 \times 108 \times 20}{513}$, or $\frac{101\frac{1}{10}}{100}$.

Ans.

51. I. 20 lbs. mutton : $16\frac{1}{4}$ lbs. salmon = price of salmon per lb. : price of mutton per lb., or price of salmon : price of mutton

$= 20 \times 4 : 65$
$= 4 \times 4 : 13$
$= 16 : 13$. Ans.

52. $\frac{2}{3} : \frac{5}{4} = 16$ days $: x$.
$\frac{5}{4} \times \frac{16}{1} \times \frac{3}{2}$, or 30 days. Ans.

53. The ratio of price is

$$\frac{16}{18} = \frac{16}{16+2} = \frac{16 \times \frac{3\frac{1}{2}}{2}}{16 \times \frac{3\frac{1}{2}}{2} + 3\frac{1}{2}}$$

$$= \frac{28}{28+3\frac{1}{2}}.$$ He must therefore have mixed the water with 28 gals. of spirits. Ans.

See Hints, p. 340.

54. I. 7236 ap. : 8442 ap. = 12s. $: x$.
$\frac{8442 \times 12}{7236}$s., or 14s. for $1\frac{1}{2}$ bus.,
and $\therefore$ for a quart
$\frac{14 \times 12}{\frac{3}{2} \times 4 \times 2 \times 4}$ d. $= 3\frac{1}{2}$d. Ans.

55. See Hints, p. 340.

56. D. $\frac{109\frac{1}{4}}{100} : \frac{115}{100} = 6\frac{1}{3}$s. $: x$.
$\frac{115 \times 19 \times 4 \times 100}{100 \times 3 \times 437}$, or 6s. 8d.
Ans.

57. D. $3 : 4 = £17\frac{11}{20} : x$.
$\frac{4 \times 351}{20 \times 3}$, or £23, 8s. Ans.

58. By Fractional Method.
Since 19s. 7d. is $\frac{235}{240}$ of £1, we must multiply the gross income by this.
$£\frac{414 \times 235}{240}$, or £405, 7s. 6d.
Ans.

59. My tax is £400 $-$ £388$\frac{3}{5}$, or £11$\frac{2}{5}$,
and $\frac{11\frac{2}{5}}{400} = £\frac{57}{5 \times 400}$,
or $\frac{57 \times 20 \times 12}{5 \times 400}$d.,
or $6\frac{2}{5}$d. in £1. Ans.

60. Since 1d. in the £1 is $\frac{1}{240}$, my gross income must be £$25\frac{1}{2} \times 240$, or £6120. Ans.

CHAPTER XIII.

1. See Hints, p. 341.

2. D. £300 : £525½ = £11¼ : *x*.

$\frac{1051 \times 45}{2 \times 4 \times 300}$, or £19, 14s. 1½d. Ans.

3. $\frac{9}{240}$ of $\frac{80}{100}$ of $\frac{1850}{1}$ = £55, 10s.

4. D. 105½ : 112 = $\frac{£26,\ 11s.\ 10\frac{3}{4}d.}{121}$: *x*

$= \frac{25531 \times 112 \times 2}{121 \times 211 \times 4}$d.

= 4s. 8d. Ans.

5. D. £6228 : £1 = £188, 2s. 9d. : *x*.

$3762\frac{9}{4} \times \frac{1}{6228}$, or $\frac{29}{48}$s., or 7¼d. Ans.

6. His tea cost him 2s. 10½d. × 72 + 2s. 6d. × 90, or $(2\frac{7}{8} \times 72 + 2\frac{1}{2} \times 90)$s., or 432s., and he receives 432s. + (72 + 90)5d.; ∴ he sells his 162 lbs. at $\frac{432 + 67\frac{1}{2}}{162}$s. a lb., or $\frac{999}{2 \times 6 \times 27}$s., or 3s. 1d. a lb. Ans.

7. £56, 13s. 9d. ~ 1s. 11¼d. × 630 = £56, 13s. 9d. ~ 1260s. − 1890 far. = £56, 13s. 9d. ~ £61, os. 7½d.; ∴ he gains £4, 6s. 10½d.

8. D. £2, 12s. 6d. : £2, 7s. 6d. = 105 : *x*.

$2\frac{3}{8} \times 105 \div 2\frac{5}{8}$,

or $\frac{19 \times 105 \times 8}{8 \times 21}$, or 95;

∴ he lost 5 per cent. Ans.

9. He paid £$\frac{23}{22}$ of $14\frac{3}{10} \times 240$, and he received £299 × 15.

D. $\frac{23}{22}$ of $\frac{143}{10} \times \frac{240}{1}$: 299 × 15 = 100 : *x*.

$\frac{299 \times 15 \times 100 \times 22 \times 10}{23 \times 143 \times 240}$,

or 125;

∴ he gained 25 per cent. Ans.

10. D. £1, 13s. 4d. : £2, 2s. 6d. = 100 : *x*.

$2\frac{1}{8} \times 100 \div 1\frac{2}{8}$,

$\frac{17 \times 100 \times 3}{8 \times 5} = 127\frac{1}{2}$,

or 27½ gain per cent. Ans.

11. 12d. : ¼d. = 100,

$\frac{1 \times 100}{4 \times 12} = 2\frac{1}{12}$.

Ans. $2\frac{1}{12}$ per cent.

12. $£4736\frac{1}{4} \times \frac{2\frac{2}{3}}{20}$,

or $\frac{18945 \times 2}{4 \times 3 \times 5}$, or £631, 10s. Ans.

13. 65×25 fr. is the debt. $\frac{65 \times 20 \times 12 \times 4}{39}$ fr. is what is paid, and $65 \times 25 \sim 5 . 20 . 4 . 4 = 25$ francs; ∴ the creditor lost 25 francs,

or $\frac{100 \times 25}{65 \times 25}$, or $1\frac{7}{13}$ per cent. Ans.

14. 1s. $11\frac{1}{2}$d. − 1s. $10\frac{1}{4}$d. = $1\frac{1}{4}$d., and $\frac{£41, 12s. 6d.}{1\frac{1}{4}d.}$ = 7992, which gives his salary in rupees. Ans. 7992 rupees.

15. $\frac{11}{12}$ of a present sov. is worth 20s. worth of pure gold; ∴ 20s. is $\frac{12}{11}$ of a present sov.; ∴ 143 pure gold sovs. = $\frac{12}{11}$ of 143 sovs., or 156. Ans.

16. Let us take 2 oranges; they cost $\frac{2}{6}$ of 2d., or $\frac{4}{6}$d., and for this he receives $\frac{1}{2}$d. + $\frac{1}{3}$d., or $\frac{5}{6}$d.

$\frac{4}{6}$d. : $\frac{5}{6}$d. = 100 : x.

$\frac{5 \times 100 \times 5}{6 \times 4} = 104\frac{1}{6}$;

∴ *he gains* $4\frac{1}{6}$ per cent. Ans.

17. 1000 : 485 = 100 : x.

$\frac{485 \times 100}{1000} = 48\frac{1}{2}$.

Ans. $48\frac{1}{2}$ per cent.

18. $£1258\frac{1}{4} \times \frac{6\frac{1}{4}}{100}$,

or $£\frac{5033}{4} \times \frac{1}{4 \times 4} = £\frac{5033}{64}$,

or £78, 12s. $9\frac{3}{4}$d. Ans.

19. 95 : 115 = £$73\frac{3}{20}$: x.

$£\frac{1463 \times 115}{20 \times 95}$, or £88, 11s. Ans.

20. 80 : 100 = £$19\frac{1}{5}$: x.

$\frac{96 \times 100}{5 \times 80}$, or £24. Ans.

21. $£35 \times 8 + £42\frac{1}{2} \times 7 - £28\frac{13}{20} \times 15$,

or $£280 + £297\frac{1}{2} - £429\frac{3}{4}$,

or £147, 15s. Ans.

22. D. £$1063\frac{3}{4}$: £518 = £1150 : x.

$£\frac{518 \times 1150 \times 4}{4255}$,

or £560. Ans.

23. $£416\frac{2}{3} \times \frac{£1}{3s. 4d.}$,

or $£\frac{1250}{3} \times 6$, or £2500. Ans.

24. His outlay was

$$3\tfrac{4\frac{1}{2}}{12}\text{s.} \times 600.$$

He received $4\frac{1}{2}$s. $\times$ 360, and $4\frac{3}{4}$s. $\times$ 81, and $3\frac{3}{8}$s. $\times$ 600

$(1620 + 303\frac{3}{4})$s. $= 100 : x$.

$$\frac{1923\frac{3}{4} \times 100}{\frac{27}{8} \text{ of } 600}, \text{ or } 95;$$

$\therefore$ he lost 5 per cent. Ans.

There is no necessity to find his actual loss, as it is not asked for.

25. He pays $£17 \times \frac{21}{20} \times 6\frac{1}{2}$.

He receives $3\frac{1}{4}$s. $\times 3 \times 112$, and

$$3\tfrac{9\frac{1}{2}}{12}\text{s.} \times 3\tfrac{1}{2} \times 112.$$

$$17 \times \tfrac{21}{20} \times \tfrac{13}{2} \times 20 : \tfrac{13}{4} \times 3 \times 112 + 3\tfrac{19}{24} \times 3\tfrac{1}{2} \times 112 = 100 : x.$$

$$\frac{112 \times (\frac{13}{4} \times 3 + \frac{91}{24} \times \frac{7}{2}) \times 100}{17 \times \frac{21}{20} \times \frac{13}{2} \times 20},$$

or $\frac{112}{1} \times \frac{1105}{48} \times \frac{100}{1} \times \frac{1}{17} \times \frac{20}{21} \times \frac{2}{13} \times \frac{1}{20}$, or $111\frac{1}{9}$;

i.e. he gains $11\frac{1}{9}$ per cent. Ans.

26. Let us take as our unit the sum he paid for the sheep. He $\therefore$ received $(\frac{115}{100}$ of $\frac{4}{9} + \frac{150}{100}$ of $\frac{1}{7} + \frac{75}{100}$ of $\frac{26}{63})$ units,

$$\text{or } \frac{322 + 135 + 195}{10.7.9},$$

$$\text{or } \tfrac{652}{630} \text{ of a unit.}$$

But $\frac{652 - 630}{630}$, or $\frac{22}{630}$ of the unit is $£3\frac{17}{20}$; $\therefore$ unit or the price of the sheep is $£\frac{77}{20} \times \frac{630}{22}$, or $£110, 5s.$ *Ans.*

27. Let each cask contain 1 gal.

He pays $12\frac{1}{2}$s. $\times 2 + 18$s. $\times 1$.

He receives 15s. $\times 3$.

$$25 + 18 : 45 = 100 : x.$$

$$\frac{45 \times 100}{43}, \text{ or } 104\tfrac{28}{43}.$$

He gains $4\frac{28}{43}$ per cent. Ans.

28. He pays $3\tfrac{7\frac{1}{4}}{12}$s. $\times$ 180.

He receives $4\frac{1}{3}$s. $\times$ 170.

$$3\tfrac{29}{48} \times 180 : \tfrac{13}{3} \times 170 = 100 : x.$$

$$\frac{13 \times 170 \times 100 \times 48}{3 \times 173 \times 180}, \text{ or } \frac{1768800}{15557}$$

$\therefore$ he gains $13\frac{859}{15557}$ per cent. Ans.

29. $160 : 100 = 3\text{s.} : x$.

The milk and water costs him $\frac{300}{100}$d., or $1\frac{7}{8}$d. per qt.

$$1\tfrac{7}{8}\text{d.} : 2\tfrac{1}{2} = 1 \text{ qt.} = x.$$

He sells each quart of milk as $\frac{5}{2} \times \frac{8}{15} = 1\frac{1}{3}$ qt. of milk and water, or he mixes $\frac{1}{3}$ of a quart. Ans.

30. He pays $£13\frac{4}{5} \times 3\frac{1}{2} + £12\frac{3}{5} \times 4$, and of this he receives altogether $\frac{112\frac{1}{2}}{100}$ of this sum, viz.,

$$£\frac{225}{2 \times 100}\left(\frac{69}{5} \times \frac{7}{2} + \frac{63}{5} \times 4\right),$$

$$\text{or } £\frac{9}{8} \times \frac{483 + 504}{5.2}.$$

$£\frac{9 \times 987}{80}$. Of this he receives for part, $£\frac{23\frac{3}{4}}{20} \times 7 \times 112$,

or $£\frac{11 \times 7 \times 112}{80}$;

$\therefore$ he sells the remaining $\frac{1}{2}$ cwt. or 56 lbs. at

$$£\frac{7(9 \times 141 - 11 \times 112)}{80},$$

$$\text{or at } \frac{7 \times 37 \times 20}{80 \times 56},$$

or 1s. $1\frac{7}{8}$d. per lb. Ans.

31. Every 2 apples cost him $(\frac{1}{2} + \frac{1}{3})$d. or $\frac{5}{6}$d., and 60d. $\div \frac{5}{6} = 72$; he therefore must sell 72×2, or 144 apples. Ans.

32. This question has been already solved in chap. VIII., ques. **27**, Key, p. 64.

33. Let us suppose he buys 12 gals. For these he pays $16\frac{1}{2}$s. $\times 7 + 10\frac{1}{2}$s. $\times 5$, and he receives of them $\frac{125}{100}$ of $(\frac{33}{2} \times \frac{7}{1} + \frac{21}{2} \times \frac{5}{1})$s.; $\therefore$ he must sell each gal. at $\frac{1}{12}$ of $\frac{5}{4}$ of $\frac{231 + 105}{2}$s., or 17s. 6d. per gal. Ans.

34. $105\frac{1}{2} : 112 = £26,\ 11s.\ 10\frac{3}{4}$d. : x.

$\frac{25531 \times 112 \times 2}{4 \times 211}$d., and this $\div 121$ will give us 56d., the *price per yard*, or 4s. 8d. Ans.

35. If we take as our unit the buying price of the 4000 quarters, the merchant receives $\frac{105}{100}$ of $\frac{1}{5} + \frac{110}{100}$ of $\frac{1}{4} + \frac{112}{100}$ of $\frac{1}{2} + \frac{116}{100}$ of $\frac{1}{20}$, or $\frac{1103}{1000}$ of the unit.

If he had sold them all at 11 per cent. profit, he would have received $\frac{1110}{1000}$ of the unit, and the difference between these is $\frac{7}{1000}$, which is worth $£72\frac{4}{5}$; $\therefore$ the cost of the corn was $\frac{364}{5} \times \frac{1000}{7}$, or £10400, and $£\frac{10400}{4000}$, or £2, 12s. gives us the buying price of each quarter. Ans.

36. His debts were $£975 \times \frac{20}{16\frac{1}{4}}$, or $£\frac{975 \times 16}{13}$. But the creditors only received $\left(\frac{82\frac{1}{2}}{100} \text{ of } \frac{3}{5} + \frac{76\frac{1}{4}}{100} \text{ of } \frac{2}{5}\right)$ of £975, or $£\frac{4}{5}$ of 975, or £780. Ans. And this would pay in the £1 $\frac{780}{1200}$ of £1, or 13s. Ans.

37. The difference between 4d. and 3d. is 1d., and this 1d. is saved by 180×4d., which he is excused; $\therefore$ his income is $£180 \times 4$, or £720. Ans.

38. Let us take as our unit the buying price of the goods.

He charges $\frac{110}{100}$ of this, but his real buying price is $\frac{1}{2} + \frac{2}{5}$

f $\frac{1}{2}$, or $\frac{7}{10}$, for which he eceives $\frac{11}{10}$.

$$\frac{7}{10} : \frac{11}{10} = 100 : x$$

$$\frac{11 \times 100 \times 10}{10 \times 7} \text{ or } 157\tfrac{1}{7};$$

$\therefore$ the true percentage is $7\frac{1}{7}$. Ans.

39.

	£	s.	d.
	3616	3	9
ubtracting	108	6	8
	3507	17	1

nd this $\div 5$ gives us the widow's hare as £701, 11s. 5d., and $\frac{99}{100}$ f this, or £694, 11s. $1\frac{23}{100}$d. ; the sisters' share, and $\frac{97}{100}$ f it, or £680, 10s. $5\frac{68}{100}$d. ; each of the nieces' share. ıfter $4\frac{1}{2}$ p. c. and the lawyer's ill of £3, 15s. have been ubtracted, there is £3449, 4s. $2\frac{7}{10}$d. left; and this must e divided in the proportion f $701\frac{1}{2}$, $694\frac{1}{2}$, and $3 \times 680\frac{1}{2}$, here would be $3437\frac{1}{2}$, and £3449, 14s. $2\frac{7}{10}$d. divided by $3437\frac{1}{2}$ gives us very little nore than £1. The numbers, herefore, would be very lightly altered from the above.

40. £35, 8s. 4d. is $£35\frac{5}{12}$. From this let us subtract the ncome tax paid on the extra £1000 at 8d. in the £1 for $\frac{1}{30}$ a year, viz. $\frac{4}{240}$ of £1000, or $£16\frac{2}{3}$.

$$£35\tfrac{5}{12} - £16\tfrac{2}{3} = £18\tfrac{3}{4}.$$

But this $£18\frac{3}{4}$ is caused by the increase of 3d. in the £1 on the original income, which is $\frac{3}{480}$ of the income. If, then, $\frac{3}{480}$ or $\frac{1}{160}$ of the income is $= £18\frac{3}{4}$, the whole income is $18\frac{3}{4} \times 160$, or £3000. Ans. The income tax is $£3000 \times \frac{5}{480} + £4000 \times \frac{8}{480}$, or $£\frac{4700}{48}$, or $£97\frac{11}{12}$. Ans.

Had the tax and income remained the same it would have been $3000 \times \frac{5}{240}$, or $£62\frac{6}{12}$, which is less than $£97\frac{11}{12}$ by $£35\frac{5}{12}$, or £35, 8s. 4d., which proves our result.

41. His original net income was $\frac{229\frac{1}{2}}{240}$ of his original gross income. But his new gross income is $\frac{9}{10}$ of $\frac{240}{229\frac{1}{2}}$ of his original net income; $\therefore$ his new net income is $\frac{235}{240}$ of $\frac{9}{10}$ of $\frac{240}{229\frac{1}{2}}$ of his old net income, and this is $= \frac{47}{51}$, or $\frac{47 \times 100}{51 \times 100}$, or $92\frac{8}{51}$; $\therefore$ his new net income is less than his old by $7\frac{43}{47}$ p.c. Ans.

42. Since 5 lbs. cost 6 sixpences, any number of lbs. $\times \frac{6}{5}$ will produce the price of the lbs. in sixpences, *e.g.* 40 lbs. will cost 48 sixpences.

43, 44. See Hints, p. 342.

45. Each egg costs me $\frac{12}{20}$d. In one case I receive for it $\frac{11}{10}$ of $\frac{12}{20}$; in the other case I receive $\frac{115}{100}$ of $\frac{12}{20}$, and 12 ($\frac{11}{10}$ of $\frac{12}{20}$ $\sim \frac{115}{100}$ of $\frac{12}{20}$)d. $= 12\left(\frac{66 \sim 69}{100}\right)$d. $= \frac{9}{25}$d. Ans.

46. ($\frac{11}{10} \sim \frac{115}{100}$) buying price per egg $= \frac{9}{25 \times 12}$d., or $\frac{3}{100}$; $\therefore$ $\frac{1}{20}$ buying price $= \frac{3}{100}$, or buying price $= \frac{6}{10}$ or $\frac{12}{20}$d., *i.e.* a shilling a score. Ans.

47. The price of each egg is $\frac{12}{20}$d., and the difference of price at different profits is $\frac{3}{100}$d., but the lesser profit is $\frac{1}{10}$ of $\frac{12}{20}$, or $\frac{6}{100}$; $\therefore$ the better profit is $\frac{6}{100} + \frac{3}{100}$, or $\frac{9}{100}$, and $\frac{12}{20} : \frac{9}{100} = 100 : x$

$$\frac{9 \times 100 \times 20}{100 \times 12} \text{ or } 15. \text{ Ans.}$$

48. This is done precisely as in **47.** The better profit is $\frac{115}{100}$ of $\frac{12}{20}$, or $\frac{9}{100}$, and $\frac{9}{100} - \frac{3}{100} = \frac{6}{100}$, and $\frac{12}{20} : \frac{6}{100} = 100 : 10$. Ans.

49. This is practically done in **46.** $(\frac{115}{100} - \frac{110}{100})$ of buying price $= \frac{9}{25 \times 12}$d.; $\therefore$ buying price $= \frac{9}{25 \times 12} \times \frac{100}{5}$d. $= \frac{12}{20}$d. or I get 20 for 1s. Ans.

50. In the former case I receive $\frac{12}{10}$d. for each egg; in the latter $\frac{12}{8}$ for each egg. $\frac{12}{8} - \frac{12}{10} = \frac{37\frac{1}{2}}{100}$ of buying price; $\therefore$ buying price $= \frac{3}{10} \times \frac{100 \times 2}{75}$, or $\frac{4}{5}$d.; $\therefore$ 100 eggs would cost $\frac{400}{5}$d., or 6s. 8d. Ans.

CHAPTER XIV.

1, 2, 3. See Hints, pp. 342, 343.

4. D. 5 ac. 3 ro. 39 po. : 35 ac. 3 ro. 34 po.
I. 12 days : 6 days
$= 4$ men : x men.

$$x = \frac{6 \times 959}{5754 \times 6 \times 4} \text{ men, or } 12 \text{ men. Ans.}$$
$$959 \times 12$$

5. I. $13\frac{5}{7}$ weeks : 4 weeks
D. £7 : £112 = 3 persons : x persons.

$$x = \frac{4 \times 112 \times 3 \times 7}{96 \times 7} \text{ persons, or } \underline{14 \text{ persons.}} \text{ Ans.}$$

6. D. 25 horses : 13 horses
D. 7 days : 60 days = £11, 6s. $0\frac{1}{2}$d. : £x.

$$x = \frac{13 \times 60 \times 5425}{25 \times 7 \times 2} \text{ pence, or } \underline{£50,\ 7s.\ 6d.} \text{ Ans.}$$

The answer given in the earlier editions is calculated for 1 days.

7. D. $7\frac{1}{2}$ bush. : 3 qrs. 6 bush.
I. 10 days : 7 days = 10 horses : x horses.

$$x = \frac{30 \times 7 \times 10 \times 2}{15 \times 10} \text{ horses, or } \underline{28 \text{ horses.}} \text{ Ans.}$$

8. D. 600 miles : 330 miles
I. 10 hrs. : 8 hrs. = 25 days : x days.

$$x = \frac{330 \times 8 \times 25}{600 \times 10} \text{ days, or } \underline{11 \text{ days.}} \text{ Ans.}$$

9. I. 17 horses : 5 horses
D. 8 bush. $1\frac{3}{4}$ pks. : 66 bush. $3\frac{3}{4}$ pks. = 9 days : x days.

$$x = \frac{5 \times 1071 \times 9}{17 \times 135} \text{ days, or } \underline{21 \text{ days.}} \text{ Ans.}$$

10. I. 24 men : 25 men
I. 5 days : 4 days = 6 hrs. : x hrs.

$$x = \frac{25 \times 4 \times 6}{24 \times 5} \text{ hours, or } \underline{5 \text{ hours.}} \text{ Ans.}$$

11. I. 25 days : 30 days
I. 8 days : 9 hrs. = 6 men : x men.
D. 1 work : 10 works

$$x = \frac{30 \times 9 \times 10 \times 6}{25 \times 8} \text{ men, or } \underline{81 \text{ men.}} \text{ Ans.}$$

12. I. 6 horses : 8 horses
D. $11\frac{2}{3}$ ac. : $17\frac{1}{2}$ ac. $= 2$ days : x days.

$$x = \frac{8 \times 35 \times 2 \times 3}{2 \times 6 \times 35} \text{ days, or } \underline{4 \text{ days.}} \text{ Ans.}$$

13. D. 9d. : 12d.
D. 12 persons : 18 persons
D. 5 wks. : 7 wks. $\Big\} = £11,\ 16s.\ 3d. : £x.$

$$x = \frac{12 \times 18 \times 7 \times 945}{9 \times 12 \times 5 \times 4} \text{s., or } \frac{1323}{2} \text{s., or } \underline{£33,\ 1s.\ 6d.} \text{ Ans.}$$

14. In the one book there are $320 \times 21 \times 11$ words, which cost £19; ∴ each word costs $£\frac{19}{320 \times 21 \times 11}$.

In the other book there are $297 \times 28 \times 10$ words, and these will cost $£\frac{19 \times 297 \times 28 \times 10}{320 \times 21 \times 11}$, or £21, 7s. 6d. Ans.

15. There are $143 \times 133 \times 9$ hours' work to do, and to do these, 171 men in 91 days would have to work

$\frac{143 \times 133 \times 9}{171 \times 91}$ hours each day,
or 11 hours. Ans.

16. To work by fractions,
8 men $\times \frac{5}{2} \times \frac{3}{2} \times \frac{3}{6}$,
or 15 men. Ans.

17. To work by fractions,
£6 $\times \frac{6}{4} \times \frac{10}{12}$, or £7, 10s. Ans.

18. To work by units.
There are $45 \times 16 \times 10$ hours' work to do, and these 100 men, working 12 hours a day, would accomplish in

$$\frac{45 \times 16 \times 10}{100 \times 12} \text{ days,}$$

or 6 days. Ans.

19. Here, again, there are $5 \times 1\frac{1}{3} \times 4\frac{1}{2}$ hours' work in the first case, and we want to find in how many days of 5 hours 1 man could do $\frac{3}{2}$ of these units of work.

This is $\frac{5 \times 1\frac{1}{3} \times 4\frac{1}{2} \times \frac{3}{2}}{5}$ days,
or 9 days. Ans.

20. Here the better way is to work by fractions, thus—

$$14 \times \frac{15}{7} \times \frac{30}{25} \times \frac{7}{2 \times 3} \times \frac{1}{2} \text{ days,}$$

or 21 days. Ans.

21. By fractions,

$5 \times \frac{5}{4} \times \frac{32}{33} \times \frac{11}{10} \times \frac{6}{5}$ days,

$= 8$ days. Ans.

22. The 50 boys could be replaced by 20 men; $\therefore$ in the second case there would be 50 men.

By fractions,

$$£60\tfrac{3}{4} \times \tfrac{32}{36} \times \tfrac{50}{54},$$

$$\text{or } £\frac{243 \times 32 \times 50}{4 \times 36 \times 54},$$

$$\text{or } £50. \text{ Ans.}$$

23. See Hints, p. 343.

3 men and 5 boys do $\frac{1}{2}$ work in 6 days, and 4 men and 6 boys do $\frac{1}{3}$ work in 3 days.

Equating the number of men in either statement,

12 men and 20 boys do 2 works in 6 days, and 12 men and 18 boys do 2 works in 6 days.

And therefore, since 20 boys do as much as 18 boys, they must be useless. Ignoring the boys, our question is, If 3 men can do half of the work in 6 days, how many can do $(1 - \frac{1}{2} - \frac{1}{3})$, or $\frac{1}{6}$ of it in 1 day?

By fractions,

$$3 \text{ men} \times 6 \times \frac{\frac{1}{6}}{\frac{1}{2}}, \text{ or } 6 \text{ men};$$

$\therefore$ 2 men must be added to the 4 to finish it in 1 day. Ans.

24. $£3 \times \frac{17}{4} \times \frac{4 \times 5\frac{1}{3}}{6\frac{1}{2}}$,

$$\text{or } £\frac{3 \times 17 \times 4 \times 16 \times 2}{4 \times 3 \times 13},$$

$$\text{or } £21, 16\text{s. } 11\tfrac{1}{13}\text{d. Ans.}$$

25. By units.

In the 1st MS. there are $6 \times 32 \times 24 \times 9$ words, and in the second $2 \times 6 \times 32 \times 24 \times 9$ words; $\therefore$ if there are 12 words in a line, and 3 sheets containing 48 pages, there must be

$$\frac{2 \times 6 \times 32 \times 24 \times 9}{12 \times 3 \times 48},$$

or 48 lines. Ans.

26. 44 labourers are equivalent to $44 \times \frac{3}{5}$ navvies.

By proportion,

I. 7 days : 15 days
I. 11 hrs. : 10 hrs.
D. 1 work : $1\frac{3}{4}$ work
$= 44 \times \frac{3}{5}$ navvies : x navvies.

$$x = \frac{15 \times 10 \times 7 \times 44 \times 3}{7 \times 11 \times 4 \times 5} \text{ navvies, or } 90 \text{ navvies. Ans.}$$

27. By fractions,

$$1\tfrac{1}{2} \text{ hrs.} \times \tfrac{1}{3} \times \tfrac{36}{3} \times \frac{1\frac{1}{3}}{1} \times \frac{1}{\frac{1}{2}},$$

$$\text{or } \frac{3 \times 36 \times 4 \times 2}{2 \times 3 \times 3 \times 3} \text{ hrs.},$$

or 16 hrs. Ans.

28. The boys that pass are $\frac{95}{100}$ of $\frac{4}{5}$ of 2500, and the girls that pass are $\frac{60}{100}$ of $\frac{1}{5}$ of 2500, or altogether ($\frac{19}{20}$ of $\frac{4}{5} + \frac{3}{5}$ of $\frac{1}{5}$), or $\frac{22}{25}$ of 2500, *i.e.* $\frac{88}{100}$ of 2500, or 88 p. c. Ans.

29. There are 4×36 hours' work in the first case, and $2 \times 4 \times 36$ hours' work in the second case. Let us take away the 6×4 hours' work done by the 6 men in the extra 4 hours. The week is $5\frac{1}{2}$ days long. Then 6 men in $5\frac{1}{2}$ days would have to work

$$\frac{11 \times 4 \times 6}{6 \times 5\frac{1}{2}} \text{ hours,}$$

or 8 hours a day. Ans.

30. Let us reduce the women and boys to men. 48 women are equivalent to 36 men, and 60 boys are equivalent to 30 men, or altogether $30+36+30$, or 96 men. In the other case there are $25+45+36$, or 106 men. By fractions we have 16 guineas $\times \frac{106}{96} \times \frac{10}{7}$, or

$$£\frac{16 \times 21 \times 106 \times 10}{20 \times 96 \times 7},$$

or £26, 10*s.* Ans.

31. Let us reduce the boys to men. 10 men and 15 boys are equivalent to $10+10$, or 20 men, and 7 men and 12 boys are equivalent to $7+8$, or 15 men. Working by fractions, 6 days $\times \frac{20}{15} =$ 8 days. Ans.

32. There are 6×11 hours' work to do, but the second men do it in $\frac{12}{13}$ of 6×11 hours, and this $\div 5\frac{1}{2}$ gives us

$$\frac{12 \times 6 \times 11 \times 2}{13 \times 11}, \text{ or } 11\tfrac{1}{13} \text{ hours.}$$

each day. Ans.

33. The ratio between the costs, since they are all direct, is $\frac{4}{3} \times \frac{5}{6} \times \frac{2}{7}$, or $\frac{20}{63}$, or 20 : 63. Ans.

34. The ratio between the times can be found thus: let $x : y$ be the ratio required.

$$\left.\begin{array}{l} \text{D. } 20 \text{ price} : 63 \text{ price} \\ \text{I. } 3 \text{ men} : 4 \text{ men} \\ \text{I. } 6 \text{ wages} : 5 \text{ wages} \end{array}\right\} = x : y$$

$$\therefore \frac{x}{y} = \frac{20.3.6}{63.4.5}, \text{ or } \frac{2}{7}. \text{ Ans.}$$

35. $4 : 3 = 60 : x$

$$\frac{3 \times 60}{4} = 45. \text{ Ans.}$$

36. If we suppose that our units are men, shillings per day, weeks, the 4 men of the first class earn with 5s. a day, in 2 weeks of 6 days each, £12. But 4 men are $\frac{4}{25}$ of 25 men, and £12 are $\frac{12}{10}$ of £10, and $\frac{4}{25} \times \frac{15}{2} = \frac{12}{10}$; $\therefore$ $\frac{15}{2}$ is the constant multiplier. Ans.

e.g. How much would 200 workmen earn? $\frac{200}{25} \times \frac{15}{2}$, or £60 × 10. The other units need not be known, as, appearing in both the numerator and denominator, they would cancel.

37. This question, as it stands, is impossible. The weekly wages cannot depend on the number of months the men work, unless it is supposed that the cost of the house is to remain unaltered. If the cost of the house remain unaltered, the multiplier would be a fraction whose numerator is a known wage, and whose denominator is a known time, and this fraction could be modified according to the units chosen.

38. Since the workmen are paid by the day, and not by the hours they work, or by the amount *or quality of* their work, we cannot take into consideration either of these two ratios. It will be found that the slower workmen receive for their work wages in the ratio of 8 : 5 compared with the others.

$$\begin{matrix} 2 \text{ gangs} : 3 \text{ gangs} \\ 4\text{s.} : 5\text{s.} \end{matrix} = x : y$$

$$\frac{x}{y} = \frac{2 \times 4}{3 \times 5} = \underline{\frac{8}{15}}. \text{ Ans.}$$

39. To solve this we must have some numbers given to us. Let them be 3 and 2 gangs of workmen, and let them take 8 and 15 units of time to finish the work. Then the houses cost

$$£\frac{3 \times 20 \times 5 \times 8 \times 310}{20},$$

$$\text{or } £\frac{2 \times 20 \times 4 \times 15 \times 310}{20},$$

$$\text{or } \underline{£37200}. \text{ Ans.}$$

40. Let us take as our unit the work per hour each man of the 2nd lot can do; then since 4 of 1st lot can do as much as 5 of the second; 1 of 1st lot can do $\frac{5}{4}$ of one of the second; and one of the 3rd lot can do as much as $\frac{2}{3}$ of one of the second; and $\therefore$ the men do $\frac{25}{4} + 7 + \frac{4}{3}$ units, and these they do in $45\frac{1}{3}$ hrs.

The second lot do $\frac{7\times5}{4}+5+\frac{3\times2}{3}$, or $\frac{63}{4}$ units. We have then this indirect proportion, $\frac{63}{4} : \frac{175}{4\times3} = 45\frac{1}{8} : x$

$$\frac{175\times136\times4}{4\times3\times3\times63}, \text{ or } \frac{3400}{81},$$

or $41\frac{79}{81}$ hrs. Ans.

41. £10 × $\frac{112}{100}$ will give me the price per cwt., and this ÷ 112 will give me the price per lb., or £$\frac{1}{10}$ or 2s. Ans.

42. $16\frac{1}{2} - 15\frac{3}{4} = \frac{3}{4}$, and 21 contains $\frac{3}{4}$, 28 times; ∴ I must have bought £2800 worth of tea. Ans.

43. Fractional method.

$100r \times \frac{100}{80} \times \frac{100}{80} \times \frac{4}{5}$, or $500r$. Ans.

Note.—We take no notice of the days and hours given, but call the two periods 100 and 80 respectively, and the powers 5 and 4.

44. The former worked 19 × 8 hours each, the latter men worked $\frac{80}{100}$ of $\frac{19\times8}{1}$, and this divided by 9 will give us the number of days they *worked, viz.* $\frac{80\times19\times8}{100\times9}$, or 13 days and $\frac{23}{45}$ of one of their days of 9 hrs. Ans.

45. To work by proportion,

ft.	ft.	
60	55	men men
days	days	= 120 : x
10	15	

$$x = \frac{55\times15\times120}{60\times10}, \text{ or } 165 \text{ men.}$$

Ans.

46. If $\frac{5625}{10000}$ of 1600 are girls who earn $\frac{9}{16}$ of the grant, and $\frac{93}{100}$ of $\frac{4375}{10000}$ of 1600 are boys who earn $\frac{7}{16}$ of the grant, we have then this proportion,

$$\frac{7}{16} : \frac{9}{16} = \frac{93\times4375}{100\times10000} : x$$

which will give us the fraction of 1600, who are girls that earn the grant.

$$\frac{9\times93\times4375\times16}{16\times100\times10000\times7}, \text{ or } \frac{837}{1600};$$

∴ 837 girls pass.

To find out what percentage this is, we have the following proportion:

$$\frac{5625\times1600}{10000} : 837 = 100 : x$$

$$x = \frac{837\times100\times10000}{5625\times1600} = 93;$$

∴ 7 per cent. of the girls failed. Ans.

47. $\frac{93}{100}$ of $\frac{5625}{10000}$ of 1600 ls, and $\frac{93}{100}$ of $\frac{4375}{10000}$ of 1600 ys earn the grant.

$\frac{5625}{4375} = \frac{225}{175} = \frac{9}{7}$;

girls earn $\frac{9}{16}$, and the boys

. Ans.

48. The girls who fail are $\frac{7}{100}$ of $\frac{5625}{10000}$ of 1600, and e boys who pass are $\frac{93}{100}$ of $\frac{4375}{10000}$ of 1600, and $\frac{7 \times 5625}{93 \times 4375}$ $= \frac{3}{31}$. Ans.

49. The number of girls who pass $\frac{93}{100}$ of $\frac{5625}{10000}$ of 1600, or 837; and then $\frac{9}{16}$ of £1440; ∴ each child earns $\frac{9 \text{ of } 1440}{16 \times 837}$, or 19s. $4\frac{8}{31}$d.

CHAPTER XV.

1, 2. See Hints, p. 344.

3. $\frac{500 \times 2 \times 4}{100} \sim \frac{500 \times 3 \times 3}{100}$,

$\frac{500}{100}$ (8∼9), or £5. Ans.

4. For 2 years at 4 per nt. the interest is $\frac{8}{100}$ of the incipal, and for 3 years at 3 r cent. is $\frac{9}{100}$, and $\frac{8 \sim 9}{100}$, or $\frac{17}{100}$ = £5; ∴ principal is ,500. Ans.

5. £5 per cent. per annum £100 is £5. 1d. in every £1 for 12 months is 1s. per nnum; and ∴ *on* £100, 100s. or £5; there is ∴ no difference.

6. $\frac{£612\frac{1}{2} \times 6 \times 3\frac{1}{2}}{100}$,

or $\frac{£1225 \times 6 \times 7}{2 \times 100 \times 2}$,

or £128, 12s. 6d.

7. £2, os. 10d. each month is £2, os. 10d. × 12, or £24, 10s. per annum, and

$\frac{24\frac{1}{2}}{612\frac{1}{2}} = \frac{49}{12250} = \frac{49}{49 \times 25} = \frac{4}{100}$;

∴ the interest is 4 per cent.

Ans.

8.

£	s.	d.
127	9	$4\frac{1}{2}$
		$3\frac{1}{8}$
382	8	$1\frac{1}{2}$
15	18	$8\frac{1}{16}$
398	6	$9\frac{9}{16}$
		$4\frac{1}{16}$
1593	7	$2\frac{1}{4}$
66	7	$9\frac{19}{32}$
£16\59	14	$11\frac{27}{32}$
20		
11\94		
12		
11\39		
32		
105		
117		
1275		

£16 11 $11\frac{1275}{3200}$,
or £16 11 $11\frac{51}{128}$. Ans.

9. The best way to do this is to find the interest on the amount for 1 year.

£	s.	d.
127	9	$4\frac{1}{2}$
		$3\frac{1}{8}$
382	8	$1\frac{1}{2}$
15	18	$8\frac{1}{16}$
3\98	6	$9\frac{9}{16}$
20		
19\66		
12		
8\01		
16		
25		

And then the following proportion will give us the time required.

£	s.	d.	£	s.	d.
3	19	$8\frac{25}{1600}$:	16	11	$11\frac{51}{128} = 1 : x$
20			20		
79			331		
12			12		
956			3983		
64			128		
3825			31864		
5736			47796		
61185			51		
2			509875		
122370					

$\frac{509875}{122370} = \frac{25}{6}$ or $4\frac{1}{6}$ yrs. Ans.

10. Here let us see what the interest on £100 for same time and at same interest would be.

$$\frac{100 \times 3\frac{1}{8} \times 4\frac{1}{6}}{100} = £\frac{625}{48}$$

$£\frac{625}{48} : £16,\ 11s.\ 11\frac{51}{128}d. = £100 : x$

$$\frac{48 \times 509875 \times 100}{625 \times 20 \times 12 \times 128},$$

or $£\frac{61185}{20 \times 12 \times 2}$, that is 61185 halfpence.

2)61185

12)30592 $\frac{1}{2}$

2,0)2549 $4\frac{1}{2}$

£127 9s.

Ans. £127, 9s. $4\frac{1}{2}$d.

11. $\frac{£1250 \times 4\frac{1}{4}}{100}$,

or $£\frac{1250 \times 17}{100 \times 4}$ or $£\frac{4250}{8}$,

or £53, 2s. 6d. Ans.

12. Since £2500 contains £100, 25 times, 25 times the interest will be £53, 2s. 6d., and $\therefore$ $53\frac{1}{8} \times \frac{1}{25}$, or $\frac{4250}{8} \times \frac{1}{25}$, or $\frac{17}{8}$, or $2\frac{1}{8}$ is the rate. Ans.

13. $£250 \times 2\frac{1}{2} \times \frac{3\frac{1}{2}}{100}$,

or $£\frac{250 \times 5 \times 7}{100 \times 2 \times 2}$, or $£\frac{175}{8}$,

or £21. 17s. 6d. Ans.

14. $\frac{250}{4}$ must be the number of £100's I possess; and $\therefore$ $£\frac{250}{4} \times 100$ must be the money I invest, or £6250. Ans.

15. $£1585\frac{7}{8} \times 6\frac{2}{3} \times \frac{4\frac{1}{2}}{100}$,

or $£\frac{12687 \times 20 \times 9}{8 \times 3 \times 2 \times 100}$, or $\frac{38061}{80}$,

that is, 38061 threepences, which are = £475, 15s. 3d.

Ans.

16. If for 8 years I receive £420, for 1 year I receive £$52\frac{1}{2}$, and $52\frac{1}{2}$ is $\frac{52\frac{1}{2}}{2000}$, or

$\frac{105}{2000 \times 2}$, or $\frac{105}{\frac{40}{100}}$, or $\frac{2\frac{5}{8}}{100}$ of £2000; $\therefore$ the interest is $2\frac{5}{8}$ per cent. Ans.

17. £1 at end of $3\frac{1}{4}$ years at 5 per cent. is worth $1 + \frac{3\frac{1}{4} \times 5}{100}$, or $1\ \frac{65}{400}$, or $\frac{465}{400}$, or $\frac{93}{80}$; $\therefore$ £3050, 10s. 6d. at the end of $3\frac{1}{4}$ years at 5 per cent. will be worth $£3050\frac{21}{40} \times \frac{93}{80}$,

or $£\frac{11347953}{40 \times 80}$,

or £3546, 4s. $8\frac{19}{40}$d. Ans.

18. $3\frac{1}{2}$ p. c. for 10 months is $\frac{6}{5}$ of $3\frac{1}{2}$ or $\frac{6}{5}$ of $\frac{7}{2}$, or $4\frac{1}{5}$ p. c. per annum. The difference between $4\frac{1}{5}$ and 4 is $\frac{1}{5}$, and $\frac{1}{5}$ is contained in £100, 500 times;

8.

£	s.	d.
127	9	$4\frac{1}{2}$
		$3\frac{1}{8}$
382	8	$1\frac{1}{2}$
15	18	$8\frac{1}{16}$
398	6	$9\frac{9}{16}$
		$4\frac{1}{16}$
1593	7	$2\frac{1}{4}$
66	7	$9\frac{19}{32}$
£16\59	14	$11\frac{27}{32}$
20		
11\94		
12		
11\39		
32		
105		
117		
1275		

£16 11 $11\frac{1275}{3200}$,
or £16 11 $11\frac{51}{128}$. Ans.

9. The best way to do this is to find the interest on the amount for 1 year.

£	s.	d.
127	9	$4\frac{1}{2}$
		$3\frac{1}{8}$
382	8	$1\frac{1}{2}$
15	18	$8\frac{1}{16}$
3\98	6	$9\frac{9}{16}$
20		
19\66		
12		
8\01		
16		
25		

And then the following proportion will give us the time required.

£ s. d. : £ s. d.
3 19 $8\frac{25}{1600}$: 16 11 $11\frac{51}{128} = 1 : x$

20	20
79	331
12	12
956	3983
64	128
3825	31864
5736	47796
	51
61185	509875
2	
122370	

$\frac{509875}{122370} = \frac{25}{6}$ or $4\frac{1}{6}$ yrs. Ans.

25. £300 contains £4, 75 times; ∴ there must be 75×100 in £7500. Ans.

26. 1d. gives an interest at 10 per cent. per annum—

$$\frac{1 \times 10}{240 \times 100}.$$

The question is, how soon will it give an interest 19s. 11d., or $£\frac{239}{240}$, which is immediately found by dividing this amount by the former, thus:—

$$\frac{239}{240} \times \frac{240 \times 100}{1 \times 10},$$

or 2390 years.

27. If £100 is $\frac{4}{100}$ of a sum of money, the sum is $£\frac{10000}{4}$, and similarly the other sum is $£\frac{10000}{5}$, and the difference between these is

$$£10000 \left(\frac{5-4}{5 \times 4}\right),$$

or £500. Ans.

28. The interest of the one is $\frac{4}{100}$ of its principal, and that of the other is $\frac{5}{100}$, and $\frac{5-4}{100} = \frac{1}{100}$; but this $\frac{1}{100}$ is the interest on £500 at 5 per cent. (£500 being the sum by which the one sum is greater than the other), or £25; ∴ the smaller principal is £2500, and this at 4 per cent. gives us £100. Ans.

29. $£\frac{1}{8} \times 80 \times \frac{4}{100}$, or the interest is $£\frac{2}{5}$ or 8s. and the amount is 2s. 6d. + 8s., or 10s. 6d. Ans.

30, 31. See Hints, p. 345.

32. Let our principal be £1. 1d. on the shilling is $\frac{1}{12}$; ∴ we must multiply the pounds by $\frac{1}{12}$. Ans.

33. $(\frac{5}{100} - \frac{4}{100})$ £1000 $= \frac{1}{100}$ of £1000, or £10; and since £10 contains £10 once, the Ans. is 1 year.

34. The interest on £1200 for 4 years at 1 per cent. is $\frac{1200 \times 4}{100}$, and on the same amount for $3\frac{1}{2}$ years at 1 per cent. is $\frac{1200 \times 3\frac{1}{2}}{100}$, and the difference between these is $1200 \times \frac{\frac{1}{2}}{100}$, or £6; and since £20 contain £6, $3\frac{1}{3}$ times, the Ans. is $3\frac{1}{3}$ per cent.

35. See Hints, p. 345.

36. 1d. per month is 1s. a year, and 1s. a year on £1 is 5 per cent. Ans.

See Hints, p. 345.

37. Let us take as our principal £100, then the interest would be 200 florins or £20; and ∴ the interest is calculated at 20 per cent. Ans.

38. $\frac{1}{4}$d. for every £1 for every day is £$\frac{100 \times 365}{4 \times 12 \times 20}$ for £100 for a year, or $38\frac{1}{48}$ per cent. Ans.

39. See Hints, p. 345.

40. At $4\frac{1}{2}$ per cent. for 10 years the invester receives $\frac{45}{100}$ of the principal. Had he invested it all at 8 per cent. for 6 years, he would have received $\frac{48}{100}$ of it, and the difference between them is $\frac{3}{100}$ of it. And this $\frac{3}{100}$ is £$\frac{200 \times 8 \times 6}{100}$ − £$\frac{200 \times 6 \times 7}{100}$, or £12; ∴ principal is £400. Ans.

41. He pays $2 \times \frac{1}{2}$d. $\times 6 + \frac{1}{2}$d. as interest, or $6\frac{1}{2}$d., and $6\frac{1}{2}$d. on 33d. is $\frac{6\frac{1}{2}}{33}$, or $\frac{13 \times 100}{33 \quad 100}$ per cent., or $19\frac{23}{33}$ per cent. for 6 months; *i.e.* $39\frac{13}{33}$ per cent. for 1 year. Ans.

42. £1 amounts to £$1 + \frac{1 \times 3\frac{1}{2} \times 2\frac{1}{2}}{100}$, or $1\frac{35}{400}$, or £$\frac{435}{400}$, or £$\frac{87}{80}$; and £1475 $\times \frac{87}{80}$, or 128325 threepences, is £1604, 1s. 3d. Ans.

And £80, 4s. $0\frac{3}{4}$d. / £1604, 1s. 3d.

$= \frac{76995}{4 \times 12 \times 20} \times \frac{16}{25665}$

$= \frac{5 \times 3 \times 5133 \times 16}{4 \times 12 \times 20 \times 5133 \times 5}$

$= \frac{5}{100}$, or 5 per cent. Ans.

43. As often as £5 is contained in £80, 4s. $0\frac{3}{4}$d., so many £100s. has he.

£80, 4s. $0\frac{3}{4}$d. $\times$ 100 / £5,

or £80, 4s. $0\frac{3}{4}$d. $\times$ 20,

or £1604, 1s. 3d.

£100 in $2\frac{1}{2}$ years at $3\frac{1}{2}$ per cent. amounts to £$\frac{435}{4}$.

£$\frac{435}{4}$: £$\frac{25665}{16}$ = £100 : x.

£$\frac{25665 \times 100 \times 4}{16 \times 435}$,

or £1475. Ans.

44. £1200 $\times \frac{5}{100}$ or 60, which is £3 more than £57, and $\frac{5}{100} - \frac{4}{100} = \frac{1}{100}$, and $3 \div \frac{1}{100}$ is £300; ∴ £300 was invested at 4 per cent., and £900 at 5 per cent.

45. We must first subtract £5, and the interest is £5 at per cent., or $5 \times \frac{7}{200}$, or $£\frac{7}{40}$.

$$£98\frac{15\frac{1}{2}}{20} - £5 - £\frac{7}{40}$$

$$= £93\frac{31}{40} - £\frac{7}{40} = £93\frac{3}{5}.$$

The question then is, what sum will amount to $£93\frac{3}{5}$ at per cent. at the end of 1 year, which is $£93\frac{3}{5} \times \frac{100}{104}$, or $£\frac{68}{5} \times \frac{100}{104}$, or £90; ∴ the smaller sum is $£\frac{90}{2}$, or £45, and the other sum is £50. Ans.

46. $£50 \times \frac{101\frac{3}{4}}{100} + £45 \times \frac{102\frac{1}{2}}{100}$, or $£\frac{407}{8} + \frac{9 \times 409}{20 \times 4}$,

or £50, 17s. 6d. + £46, 0s. 3d., or £96, 17s. 9d. Ans.

47. £1 in 8 years at 20 per cent. amounts to £2·60, which we must divide into 42998·1696.

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26 )42998·1696( 16537·7575
   ---
   169
   156
   ---
   139
   130
   ---
    98
    78
    --
   201
   182
   ---
   196
   182
   ---
   149
   130
   ---
   196
   182
   ---
   140
```

Ans. £16537·7575, etc.

48. $\frac{£56\frac{1}{4} \times 1 \times 3\frac{1}{2}}{100}$ is the interest at 1 per cent., and the number of times £64, 2s. 6d. − £56, 5s. 0d., or £7, 17s. 6d. contains this interest will give us the rate we want.

$$\frac{63}{8} \times \frac{100 \times 4 \times 2}{225 \times 7},$$

or 4 per cent. Ans.

49. £500 will produce in 3 years at 5 per cent. per annum $£\frac{500 \times 3 \times 5}{100}$, or £75.

$£\frac{1 \times 5\frac{61}{240} \times 15}{100}$ will give us the interest produced by £1, and £75 ÷ by this, will give us the principal required.

$\frac{75 \times 100 \times 240}{1261 \times 15}$, or $£95\frac{205}{1261}$. Ans.

50. See Hints, p. 346.

CHAPTER XVI.

1. See Hints, p. 346.

2. £100 is worth £108 2 years hence at 4 p. c.
Ans. £100.

3. See Hints, p. 346.

4. Since £1 is the interest on £100 for 4 months at 3 p. c., £1 is the discount on £101; ∴ the following proportion will give us the discount.

£101 : £252, 10s. = £1 : £x

$$x = \frac{252\frac{1}{2}}{101}, \text{ or } \frac{505}{2 \times 101},$$

or £2, 10s. Ans.

5. The interest on £100 at 4 p. c. for 3 months is £1; ∴ £1 is the discount on £101; ∴ if 15s. $10\frac{10}{101}$d. be the discount, the sum discounted is 15s. $10\frac{10}{101}$d. × 101, or $\frac{19200}{101}$d. × 101,
or £80, os. od. Ans.

6. £100 at 5 p. c. for 4 months will be worth

$$£100\frac{5 \times 4}{12}, \text{ or } £101\tfrac{2}{3};$$

∴ £$1\frac{2}{3}$ is the discount, or £$101\frac{2}{3}$ similarly. £100 at 6 p. c. for 4 months will be worth $£100\frac{6 \times 4}{12}$, or £102. If then the discount be £5 when discounted at 6 p. c., to find the other we have this proportion:

$$\frac{2}{102} : \frac{1\frac{2}{3}}{101\frac{2}{3}} = £5 : x$$

$$\frac{5 \times 3 \times 5 \times 102}{3 \times 305 \times 2} = £4, 3s. 7\tfrac{17}{61}d.$$

Ans.

7. 1s. 6d. is the interest on the discount at 4 p. c. for 3 months; ∴ the discount is $£\frac{100 \times 1\frac{1}{2}}{4 \times \frac{1}{4} \times 20}$, or £7, 10s., and the principal, of which £7, 10s. is the discount at 4 p. c. for 3 months, is £$7\frac{1}{2} \times \frac{101}{1}$, or £757, 10s. Ans.

8. £1 is the interest on the discount at 3 p. c. for 6 months; ∴ the discount is $£\frac{100 \times 1}{3 \times \frac{1}{2}}$, or £$66\frac{2}{3}$; and this is the discount on a sum given by the following proportion:

£$1\frac{1}{2}$: £$66\frac{2}{3}$ = £$101\frac{1}{3}$: £x

$$x = £\frac{200 \times 203 \times 2}{3 \times 2 \times 3},$$

or £4511, 2s. $2\frac{2}{3}$d.

9. This can be shown by mbols, thus:

$$I = \frac{Prt}{100}.$$

$$D = \frac{Prt}{100 + rt}.$$

$$I - D = Prt\left(\frac{1}{100} - \frac{1}{100+rt}\right)$$

$$= \frac{Prt}{100}\left(\frac{rt}{100+rt}\right)$$

$$= \frac{\left(\frac{Prt}{100+rt}\right)rt}{100} = \frac{Drt}{100}$$

hich is the interest on the scount. See par. 10 of this apter.

10. See Hints, p. 346.

11. The interest on £100 r 4 months at 1 p. c. is £$\frac{1}{3}$; . the discount on £100$\frac{1}{3}$ for months at 1 p. c. is £$\frac{1}{3}$. f the discount is £1, the nterest is calculated at £1 - £$\frac{1}{3}$, or 3 p. c. Ans.

12. Since $\frac{1}{21} = \frac{5}{105}$; ∴ rate 5 p. c., or $\frac{1}{20}$ of this sum. Ans.

13. $\frac{1}{10} = \frac{10}{100}$; ∴ the multi-lier will be $\frac{10}{110}$, or $\frac{1}{11}$. Ans.

14. The interest is $\frac{20}{1000}$, r $\frac{2}{100}$; ∴ the discount will e $\frac{2}{102}$, or $\frac{1}{51}$ of £1000, *i.e.* £19, 12s. $1\frac{45}{51}$d. Ans.

15. This example, I am sorry to find, cannot be worked by pure arithmetic, since an unknown quantity is involved as a factor in a term. For $\frac{1}{420}$ let us read $\frac{1}{30}$. At 4 p. c. for 1 year the difference is $\frac{4}{100} - \frac{4}{104}$, or $4 \times \frac{4}{100 \times 104}$P; similarly for 2 years the difference is $2 \times 4 \times \frac{2 \times 4}{100 \times 108}$P, and generally the difference for t years is

$$t4 \times \frac{t \times 4}{100 \times (100 + t \times 4)}\text{P},$$

and if $\frac{1}{30}\text{P} = \frac{t^2 16}{10000 + 400t}\text{P}$, we get $t = 5$ years. Ans.

16. The interest on the discount is 25s. $2\frac{2}{5}$d. − 24s. $9\frac{27}{29}$d., or $4\frac{68}{145}$d.;

$$\therefore r = \frac{100 \times 4\frac{68}{145}}{£1,\ 4\text{s. } 9\frac{27}{29}\text{d.} \times \frac{1}{2}}$$

$$= \frac{100 \times 648 \times 2}{8640 \times 5},$$

or 3 p. c. Ans.

We can at once find the principal from the interest.

$$P = \frac{100 \times 25\text{s. } 2\frac{2}{5}\text{d.}}{3 \times \frac{1}{2}}$$

$$= \frac{100 \times 1512 \times 2}{20 \times 12 \times 5 \times 3}$$

$$= £84. \text{ Ans.}$$

17. 2s. is the interest on the discount for 4 months at 4 p. c.

$$\text{Discount, } \frac{100 \times 2s.}{4 \times \frac{1}{3}},$$

$$\text{or } \frac{100 \times 2 \times 3}{4 \times 20}, \text{ or } £7, 10s.$$

$\therefore$ the interest is £7, 12s. and the principal is

$$£\frac{100 \times 152}{4 \times \frac{1}{3} \times 20}, \text{ or } \underline{£570}. \text{ Ans.}$$

18. See Hints, p. 347.

19. The interest at 3 p. c. for 6 months is $\frac{3 \times \frac{1}{2}}{100}$ of the sum, and that at 4 p. c. for same time is $\frac{4 \times \frac{1}{2}}{100}$ of the sum.

$$\text{or } \frac{\frac{2}{100} - \frac{3}{200}}{200} \text{ of sum} = £\frac{13}{10};$$

$\therefore$ the sum is $\underline{£260}$. Ans.

20. The discounts at 4 and 6 p. c. for 6 months are respectively $\frac{\frac{1}{2} \times 4}{100 + \frac{1}{2} \times 4}$ and $\frac{\frac{1}{2} \times 6}{100 + \frac{1}{2} \times 6}$ of the sum discounted and the difference between them, or $\frac{3}{103} - \frac{2}{102}$ of the sum $= £\frac{5000}{1751}$;

$$\therefore \text{ sum} = £\frac{5000}{1751} \times \frac{103 \times 102}{100}$$

$$= \underline{£300}. \text{ Ans.}$$

21. The present worth is

$$£276, 10s. 5d. \times \frac{100}{100 + \frac{219}{365} \times \frac{7}{2}},$$

$$\text{or } £\frac{66365}{240} \times \frac{100 \times 5 \times 2}{1021}, \text{ or}$$

66500d., or $\underline{£270, 16s. 8d.}$ Ans.

22. This is just the converse of the last, and can be done thus:

$$£\frac{65000}{240} \times \frac{100 + \frac{219}{365} \times \frac{7}{2}}{100},$$

$$\text{or } £\frac{65000}{240} \times \frac{1021}{100 \times 5 \times 2},$$

or 66365d., or $\underline{£276, 10s. 5d.}$ Ans.

23.

$$£328, 13s. 5d. \times \frac{\frac{1}{4} \times 4}{100 + \frac{1}{4} \times 4},$$

or £328, 13s. 5d. $\times \frac{1}{101}$, which is best done by division.

£	s.	d.	£
101) 328	13	5 (	3
303			
25			
20			
513 (	5s.		
505 (			
8			
12			
101 (	1d.		
101 (			

Ans. $\underline{£3, 5s. 1d.}$

24. $\frac{£3,\ 5s.\ 1d.}{£328,\ 13s.\ 5d.} = \frac{781}{78881}$ or $\frac{1}{101}$; and since this is a fraction whose terms differ by 100, the rate must be $1 \div \frac{1}{4}$ (the time), or 4 p. c. Ans.

25.

£	s.	d.
3	28	13
	20	
	573s.	
	12	
	8	81d.

$5 \times \frac{4 \times \frac{1}{4}}{100}$

	£	s.	d.
or	3	5	$8\frac{81}{100}$
Subtracting	3	5	1
			$7\frac{81}{100}$

	£	s.	d.
The interest of	3	5	1
	20		
	65s.		
	12		
	7	81d.	

is $7\frac{81}{100}$d., which proves what we wanted to.

26. $£650 \times \frac{5 \times \frac{1}{4}}{100}$,

or $\frac{650}{1} \times \frac{5}{4 \times 100}$, or £8, 2s. 6d.

The discount on £495, 12s. 6d. due in 4 months at 5 p. c. is

$£495\frac{5}{8} \times \frac{\frac{1}{3} \times 5}{100 + \frac{1}{3} \times 5}$,

or $£\frac{3965}{8} \times \frac{1}{61}$, or £8, 2s. 6d. Ans.

Which proves the statement.

27. The interest and discount are $\frac{4 \times \frac{1}{4}}{100}$ and $\frac{4 \times \frac{1}{4}}{100 + 4 \times \frac{1}{4}}$ respectively of the sum, and $\frac{1}{100} - \frac{1}{101}$ is 12s.; ∴ the sum is 12s. × 100 × 101, or £6060. Ans.

28.

$£929,\ 10s \times \frac{2\frac{1}{4} \times 2\frac{1}{2}}{100 + 2\frac{1}{4} \times 2\frac{1}{2}}$,

or $£\frac{1859}{2} \times \frac{9 \times 5}{845}$, or £49, 10s. Ans.

29. $\frac{£49,\ 10s.}{£929,\ 10s.} = \frac{99}{1859}$ $= \frac{9}{169}$, and we have to reduce this to a fraction whose terms differ by 100.

$$\frac{9}{9 + 160} = \frac{9 \times \frac{10}{16}}{9 \times \frac{10}{16} + 100};$$

∴ $9 \times \frac{10}{16}$, or $\frac{45}{8}$, is the rate × time; ∴ time is $\frac{45}{8} \div 2\frac{1}{2}$, or $2\frac{1}{4}$ years. Ans.

30. The discount is

$$\frac{4\frac{1}{4} \times \frac{4\frac{1}{2}}{12}}{100 + 4\frac{1}{4} \times \frac{4\frac{1}{2}}{12}}$$ of the sum,

or $\frac{\frac{51}{32}}{101\frac{19}{32}}$, or $\frac{51}{3251}$ of the sum, and this is worth £11, 13s. 9d.; ∴ the sum is £11, 13s., 9d. $\times \frac{3251}{51}$,

or $\frac{2805 \times 3251}{51}$ pence,

or $\frac{3039685}{17}$ pence,

or £745, os. 5d. Ans.

31. See Hints, p. 347.

32. We have to find the present worth of £500 due 6 months hence at 5 per cent., and £500 due 12 months hence at 5 per cent., which are

$$£500 \times \frac{100}{100+5} + £500 \times \frac{100}{100+\frac{5}{2}},$$

or $£500 \times (\frac{20}{21} + \frac{40}{41})$,

or $£500 \times \frac{1660}{21 \times 41}$,

or £963, 19s. $10\frac{254}{287}$d. Ans.

33. £36, os. $1\frac{33}{287}$d. is the discount, and $\frac{£36,\ \text{os. } 1\frac{33}{287}\text{d.}}{£1000}$ is a fraction whose terms must be reduced to differ by 100, viz. $\frac{31 \times \frac{100}{830}}{31 \times \frac{100}{830} + 100}$;

$\therefore 5t = \frac{310}{83}$

or $t = \frac{62}{83}$ *of a year.* Ans.

34. $£\frac{450 \times 4 \times \frac{7}{12}}{100}$,

$£\frac{450 \times 4 \times 7}{1200}$, or £10; 10s.,

and £460, 10s. $\times \frac{4 \times \frac{7}{12}}{100 + 4 \times \frac{7}{12}}$

$= £460\frac{1}{2} \times \frac{7}{307}$,

or $£\frac{921 \times 7}{2 \times 307}$

or £10, 10s., the same as before.

35. See Hints, p. 347.

36. £1 at this rate and for this time will amount to

$$1 + \frac{3\frac{1}{4} \times 10}{100}, \text{ or } £\frac{53}{40};$$

∴ £83, 17s. $1\frac{17}{40}$d. $\times \frac{40}{53}$ is the sum required, and this can easily be shown to be 15189d., or £63, 5s. 9d. Ans.

37. The difference is

$$£100 \left\{ \frac{5 \times 5}{100} - \frac{5 \times 5}{125} \right\},$$

or £100 $(\frac{1}{4} - \frac{1}{5})$, or £5. Ans.

38. $\frac{£200}{£218} = \frac{100}{100+9}$;

∴ 9 is the time × rate, but the time is 2 years; ∴ the rate is $4\frac{1}{2}$ per cent. Present worth of

1000 due 6 years hence at is

$$£1000 \times \frac{100}{100+27},$$

$$\text{or } £\frac{100000}{127},$$

£787, 8s. $0\frac{48}{127}$d. Ans.

39.

£	s.	d.
793	14	$10\frac{1}{2}$

$\times \frac{4 \times \frac{3}{4}}{103}$

3

103)2381 4 $7\frac{1}{2}$(23
206

321
309

12
20

244(2
206

38
12

463($4\frac{1}{2}$
412

$51\frac{1}{2}$
$51\frac{1}{2}$

Ans. £23, 2s. $4\frac{1}{2}$d.

40. The present value is found by subtracting the discount.

£	s.	d.
793	14	$10\frac{1}{2}$
23	2	$4\frac{1}{2}$
770	12	6

And the amount of this 9 months hence at 4 per cent. is

$1 + \frac{4 \times \frac{3}{4}}{100}$, or $\frac{103}{100}$ of it.

£	s.	d.
770	12	6
		103

100)79374 7 6
20

14887s.
12

11850d.

£793, 14s. $11\frac{1}{2}$d. Ans.

41. $£237\frac{1}{2} \times \frac{2 \times 7}{114}$;

or $£\frac{475 \times 7}{2 \times 57}$, or $£\frac{175}{6}$,

or £29, 3s. 4d. Ans.

42. $£10 \times \frac{10 \times 1}{110}$, or $£\frac{10}{11}$,

or 18s. $2\frac{2}{11}$d. Ans.

43. The discount : interest $= P\,\frac{r \times t}{100 + r \times t} : P\,\frac{rt}{100}$,

but $\frac{1}{100 + rt} : \frac{1}{100} = 50 : 53$;

$\therefore 100 : 100 + rt = 50 : 53$

$= 100 : 100 + 6$;

$\therefore \underline{rt = 6}$. Ans.

44. $\frac{4 \times \frac{15}{12}}{100 + 4 \times \frac{15}{12}}$,

or $\frac{5}{100 + 5}$, or $\underline{\frac{1}{21}}$. Ans.

45. $\frac{100}{100 + \frac{1}{4} \times 6}$, or $\frac{400}{406}$,

or $\underline{\frac{200}{203}}$. Ans.

46. $1 + \frac{2 \times 3}{100}$, or $\frac{106}{100}$,

or $\underline{\frac{53}{50}}$. Ans.

47. $2120 \times \frac{50}{53}$, or $\underline{£2000}$. Ans.

48. The principal is

$\frac{100 \times 18\text{s. } 7\frac{797}{900}\text{d.}}{4\frac{1}{2} \times \frac{7}{12}}$,

or $\frac{100 \times 223\frac{797}{900}}{\frac{9}{2} \times \frac{7}{12}}$ pence,

or $\frac{100 \times 201497 \times 2 \times 12}{900 \times 9 \times 7}$ pence,

or $£\frac{201497 \times 2 \times 4}{3 \times 9 \times 7 \times 20 \times 12}$,

and the discount on this for the same time at the same rate is $£\frac{201497}{3 \times 9 \times 7 \times 5 \times 6}$

$\times \frac{4\frac{1}{2} \times \frac{7}{12}}{100 + 4\frac{1}{2} \times \frac{7}{12}}$,

or $£\frac{201497}{3 \times 9 \times 7 \times 5 \times 6} \times \frac{21}{821}$,

or $\underline{18\text{s. } 2\frac{1174}{7385}\text{d.}}$ Ans.

49. The amount of £124, 13s. 4d. for $7\frac{1}{2}$ years at 5 per cent. is $£124\frac{2}{3} \times \frac{100 + 7\frac{1}{2} \times 5}{100}$,

or $£\frac{374 \times 11}{3 \times 8}$, or $£\frac{187 \times 11}{3 \times 4}$,

and the discount on this for the same time and at the same rate is $£\frac{187 \times 11}{3 \times 4} \times \frac{37\frac{1}{2}}{137\frac{1}{2}}$,

or $£\frac{187 \times 11 \times 3}{3 \times 4 \times 11}$,

or $\underline{£46, 15\text{s.}}$ Ans.

50. The multiplier will be

$\frac{\frac{1}{12} \times 3}{100 + \frac{1}{12} \times 3}$, or $\frac{\frac{1}{4}}{100\frac{1}{4}}$,

or $\underline{\frac{1}{401}}$. Ans.

The multiplier for 2 months would be $\frac{\frac{1}{6} \times 3}{100 + \frac{1}{6} \times 3}$, or $\frac{\frac{1}{2}}{100\frac{1}{2}}$,

or $\frac{1}{201}$, which is less than double $\frac{1}{401}$.

CHAPTER XVII.

1. $\frac{3}{3+5}=\frac{3}{8}=\frac{3\times3}{8\times3}=\frac{9}{24}$.
Ans.

2. $\frac{3}{8}$ of $48=18$ lbs.,
$\frac{5}{8}$ of $48=30$ lbs.;
Ans. 18 lbs. and 30 lbs.

3. $\frac{\frac{1}{2}}{\frac{1}{2}+\frac{1}{3}}$, or $\frac{3}{3+2}$ of $100=60$,
and $\frac{2}{3+2}$ of $100=40$.
Ans. 60 and 40.

4. See Hints, p. 348.

5. $\frac{2}{3}:\frac{5}{7}=14:15$.
$\frac{14}{29}$ of $203=98$,
$\frac{15}{29}$ of $203=105$.
Ans. 98 and 105.

6. $\frac{4}{9}$ of $180=80$,
$\frac{5}{9}$ of $180=100$,
and (80×5)s. $=(100\times4)$s.;
∴ 100 lbs. of 4s. tea must be mixed with 80 lbs. of 5s. tea. Ans.

7. 1s. 1d. $=13$,
1s. 3d. $=15$;
∴ $\frac{13}{13+15}$ of £105
$=$ £48, 15s.,
and $\frac{15}{28}$ of £105 $=$ £56, 5s.
Ans. £48, 15s. and £56, 15s.

8. $15\frac{1}{2}\times12=186$,
and $186-149=37$,
and $\frac{37}{12-8}=9\frac{1}{4}$.
Ans. $9\frac{1}{4}$ and $6\frac{1}{4}$.

9. £10 $\times$ 13 $=$ £130,
£130 $-$ £120 $=$ £10,
$\frac{\text{£}10}{13-11}=$ £5.
Ans. £5 and £5.

10, 11. See Hints, p. 348.

12. $25\times3=75$,
$75+2=77$,
$\frac{77}{3+4}=11$;
∴ numbers are 11 and 14.
Or, $25\times4=100$,
$100+2=102$,
$\frac{102}{3+4}=14\frac{4}{7}$.
Other Ans. $14\frac{4}{7}$ and $10\frac{3}{7}$.

13, 14, 15, 16. See Hints, p. 348.

17. $16\times8=128$,
$128-100=28$,
$\frac{28}{8-5}=9\frac{1}{3}$.
Ans. $9\frac{1}{3}$ and $6\frac{2}{3}$.

18, 19. See Hints, p. 349.

20. See Hints, p. 349. 8 times the smaller part = 2 ; ∴ it is $\frac{1}{4}$, and the other part is $9\frac{3}{4}$. Ans.

21. 2s. 8d. − 2s. 6d. = 2d., and 2s. 9d. − 2s. 8d. = 1d. ; ∴ the teas must be mixed in the ratio of 1 : 2, or twice as many lbs. of the dearer tea.

22. $\frac{£15,\ 15s.}{150} = \frac{21}{10}s.$,

$2\frac{1}{6}s. - \frac{21}{10}s. = \frac{1}{15}s.$,

and $\frac{21}{10}s. - 2s. = \frac{1}{10}s.$;

∴ the tea must be mixed in the ratio of $\frac{1}{10}$ to $\frac{1}{15}$, or 3 : 2 ;

∴ $\frac{2}{5}$ of 150, or

60 lbs is worth 2s. Ans.

23. 17s. − 16s. 6d. = 6d., 18s. 6d. − 17s., or 1s. 6d. ; ∴ the brandies are mixed in the ratio of 3 : 1, or 3 of the cheaper to 1 of the dearer. Ans.

24. In the adulterated brandy there is $\frac{15}{16}$ of brandy.

$$1 - \frac{15}{16} = \frac{1}{16},$$
$$1 - \frac{31}{32} = \frac{1}{32};$$

and since $\frac{1}{32}$ is $\frac{1}{2}$ of $\frac{1}{16}$, ∴ the brandies must be mixed *in equal* quantities, or 1 gal. *of adulterated with* 1 gal. of *pure. Ans.*

25. The buying price is $\frac{85}{100}$ of $\frac{90}{100}$ of 25d. + $\frac{10}{100}$d., or $\frac{769}{40}$d., and $\frac{769}{40}$d. : 1s. 10d. = 100 : x.

$$x = \frac{22 \times 100 \times 40}{769} = 114\frac{334}{769},$$

or $14\frac{334}{769}$ p. c. Ans.

26. 3 : 5 = 3 : 5 ; ∴ the arms are 3 ft. and 5 ft.

Ans. 8 ft.

27. 10 times the power = 2 × 100 ; ∴ the power is 20 lbs. Ans.

28. If the pressure on the fulcrum is 20 lbs., the power + the weight = 20, and these must be in the ratio of 5 : 7. $\frac{5}{12}$ of 20 = $\frac{25}{3}$, or $8\frac{1}{3}$ lbs. Ans.

29. The power × 100 = 3 × 1000 lbs. ; ∴ the power = 30 lbs. Ans.

30. The student is recommended to draw a figure in every case. Let us take moments round the point where the thinner end of beam rests on man's shoulder, then upward pressure of other man's shoulder × 30 = 100 × 21 ; ∴ the man nearer the heavier end bears $\frac{100 \times 21}{30}$, or 70 lbs., and the other 30 lbs. Ans.

31. See Hints, p. 349.

32. Using the answer of 31, nd taking moments round A, re have the pressure on the ulcrum $\times 5 = 4 \times 2 + 2 \times 3 + 101 \times 4 + 78\frac{1}{2} \times 7$, and this $\div 5$ gives us $193\frac{1}{2}$ lbs. Ans.

33. Taking moments round he fulcrum, the force required $\times 3 = 4 \times 3 + 2 \times 2 + 101 \times 1 - 8 \times 5$, and this $\div 3$ gives us $5\frac{2}{3}$ lbs. Ans.

34. If the pressure on the ulcrum be 17 lbs., the weight nust be 20 lbs. (not 22 as rinted in earlier editions). Taking moments round B, we ave 20 × the distance $= 5 \times 6 + 17 \times 4 - 2 \times 3$, and this $\div 20$ ives us $\frac{23}{5}$, or $4\frac{3}{5}$ feet from B, *i.e.* $1\frac{2}{5}$ feet from A. Ans.

35. The pressure on the ulcrum $= 4 + 10 + 10$ lbs., or 4 lbs.; and taking moments ound the end where the 10 bs. hang, we have 24 × the listance from the 10 lbs. $= 4 \times 6 + 10 \times 3$, and this $\div 24 = 2\frac{1}{4}$; ∴ the fulcrum must be $\frac{1}{4}$ ft. from where the 10 lbs. ang, or $\frac{3}{4}$ ft. from the middle f lever. Ans.

36. If the lever will balance ft. from A, we may consider ts weight to act at that point. Taking *moments round the point 2 ft. from B, the weight* of the lever $\times 1 = 10 \times 2$; ∴ the weight is $= 20$ lbs. Ans.

37. The pressure on the fulcrum is 109 lbs. Taking moments round B, we have $109 \times 4 = 1 \times$ length of the lever $+ 8 \times$ half the length of the lever, or $4 \times$ length of the lever; ∴ the length of the lever $= \frac{109 \times 4}{5}$, or $87\frac{1}{5}$ ft. Ans.

38. See Hints, p. 349.

39. This is dividing 10 into two parts in the ratio of 4 to 5.

$\frac{4}{9}$ of $10 = 4\frac{4}{9}$,

and $\frac{5}{9}$ of $10 = 5\frac{5}{9}$;

∴ $4\frac{4}{9}$ and $5\frac{5}{9}$ are the parts. Ans.

40. $7 \times 17 = 119$,

and $119 - 2 = 117$,

and $\frac{117}{7+6} = 9$;

∴ the parts are 9 and 8. Ans.

41. $7 \times 17 = 119$,

$119 + 2 = 121$,

$\frac{121}{7+6} = 9\frac{4}{13}$;

∴ the two parts are $9\frac{4}{13}$, $7\frac{9}{13}$. Ans.

42. $6 \times 17 = 102$,

$102 + 12 = 114$,

$\frac{114}{7+6} = 8\frac{10}{13}$;

∴ the two parts are $8\frac{10}{13}$, $8\frac{3}{13}$. Ans.

43. $\frac{1}{25}-\frac{1}{28}=\frac{3}{25\times 28}$,

$\frac{1}{28}-\frac{1}{30}=\frac{1}{28\times 15}$;

∴ I must mix them in the ratio $\frac{1}{28\times 15}:\frac{3}{25\times 28}$,

or $25\times 28 : 28\times 15\times 3$,

or 5 : 9. Ans.

This may be proved thus—

$\frac{1\times 5}{25}+\frac{1\times 9}{30}=\frac{1\times(5+9)}{28}$,

or $\frac{1}{5}+\frac{3}{10}=\frac{1}{2}$.

44. We must divide them in the ratio of 12 : 1.

$\frac{12}{13}$ of £317, 4s. = £292, 16s.,

$\frac{1}{13}$ of £317, 4s. = £24, 8s.

Ans.

45. The boys have $\frac{2}{5}$ and the girls $\frac{3}{5}$ of £250, or £100 and £150. Ans.

46. Each of the boys received $\frac{£100}{\frac{2}{5}\text{ of }100}$, or £$\frac{5}{3}$.

Similarly, each of the girls receives $\frac{£150}{\frac{2}{5}\text{ of }100}$, or £$\frac{15}{4}$,

and £$\frac{5}{3}$: £$\frac{15}{4}$ = 4 : 9. Ans.

47. The weight of the beam acts 2 ft. from the fulcrum. Then the weight of the beam $\times 2 = 112\times 6$;

∴ the weight of beam is 112×3, or 336 lbs. Ans.

48. See Hints, p. 350.

49. The arms must be as 2 : 10, or the arms must be $\frac{1}{6}$ and $\frac{5}{6}$ of 10 ft., or $1\frac{2}{3}$ ft. and $8\frac{1}{3}$ ft. Ans.

50. See Hints, p. 350.

CHAPTER XVIII.

1. £1 is worth $1\frac{6}{100}$, or £1·06; ∴ the principal × 1·06 will give us the amount at end of the 1st year, and this product × 1·06 will give us the amount at end of 2nd year, and so on. Ans.

2.

£250
1·06
1500
250
265·00
1·06
1590
265
£280·90

Ans. £280, 18s.

The amount can be
l as in Ques. 2; hence
nterest is £30, 18s. Ans.

£270
1·03
810
270
£278·1⅘
1·03
8343
2781
£286·443
20
8·86⅘s.
12
10·32d.

ns. £286, 8s. $10\frac{8}{25}$d.

12)3·d.
2⅟0)11·25s.
£1397·5625
1·04
55902500
13975625
1453·465⅘⅘⅘
1·04
5813860
5813860
1453465
1511·6036⅘
1·04
60464144
15116036
1572·067744
1397·5625
£174·505244
20
10·10488⅘s.
12
1·25856d.

Ans. £174, 10s. $1\frac{808}{3125}$d.

6. £1397, 11s. 3d.
= £1397·5625,
and this × 1·04 twice
= £1511·6036
20
12·072⅘s.
12
·864d.

Ans. £1511, 12s. $0\frac{108}{125}$d.

7. This is best done by fractions.

$$£157\,2\frac{1\frac{1}{3}}{20} \div 1\tfrac{4}{100} \div 1\tfrac{4}{100} \div 1\tfrac{4}{100} = £\tfrac{23581}{15} \times \tfrac{100}{104} \times \tfrac{100}{104} \times \tfrac{100}{104} = £1397,\ 11s.\ 2\tfrac{1692}{2197}d.,$$

or very nearly £1397, 11s. 3d.

8. £512
$6\frac{1}{4}$
3072
128
32·00
512
£544·amount for 1st year.
$6\frac{1}{4}$
3264
136·
34·00
544
£578 amount for 2nd year.
$6\frac{1}{4}$
3468
$144\frac{1}{2}$
$36·12\frac{1}{2}$
578
£614·125 amount for 3rd year.
20
2·5ṡ
12
6·ḋ.
Ans. £614, 2s. 6d.

9. See Hints, p. 350.

10. £2500
1·045
12500
10000
2500
2612·5ṡ
1·045
130625
104500
26125
2730·0625
1·045
136503125
109202500
27300625
£2852·9153125
20
18·30625ṡ.
12
3·675ḋ.
Ans. £2852, 18s. $3\frac{27}{40}$d.

11. See Hints, p. 350.

We must first reduce t amount given to £'s decimal and then divide this by 1·0 three times, when we sh obtain £2500. Ans.

12. See Hints, p. 350.

5)2·0 fifths

12)2·4d.

20)13·2s.

1·04)699·66(672·75
624

756
728

286
208

780
728

520
520

...

1·035)672·750(650
6210

5175
5175

Ans. £650.

13. 1·0475 having so many figures, it will be better to work by fractions.

£1272
·04¾

5088
954

60·42

60·42
1272

£1332·42
·04¾

532968
999315

63·28995
1332·42

£1395·70995
·04¾

55828380
10467824625

66·296222625
1395·70995

£1462·006172625
£1272

£190·006172625

Ans. £190, os. $1\frac{48143}{100000}$d.

The fraction is nearly $= \frac{1}{2}$.

14. $£1000 \times \frac{102\frac{1}{2}}{100} \times \frac{102\frac{1}{2}}{100}$

will give us the amount, viz.

$$\frac{1000 \times 205 \times 205}{100 \times 100 \times 2 \times 2},$$

and this less £1000 will give us the interest, viz.

£50, 12s. 6d. Ans.

15. $£1200 \times \frac{105}{100} \times \frac{105}{100} \times \frac{105}{100} \times \frac{105}{100}$

$= £\frac{12 \times 21 \times 21 \times 21 \times 21}{2000}$

$= £\frac{583443}{400}$

$= £1458,\ 12s.\ 1\frac{4}{5}d.$ Ans.

16.

£2000
1·075

£2150·
1·075

10750
15050
2150

£2311·25
1·075

1155625
1617875
231125

£2484·59375
20

11·875s.
12

10·500d.

Ans. £484, 11s. 10½d.

17. The difference will be the interest on the simple interest for 1 year.

$£2718\frac{3}{4} \times \frac{3}{100} \times \frac{3}{100}$,

or $£\frac{10875 \times 3 \times 3}{4 \times 100 \times 100}$,

or £2, 8s. 11¼d.

18. See Hints, p. 351.

19.

£	s.	d.
113	5	$10\frac{4}{5}$

20

2265s.

12

27190·8d.;

and this we have to multipl (i.) by $\frac{100}{105}$, and (ii.) by $\frac{100}{103\frac{3}{4}}$ or $\frac{80}{83}$.

105)2719080·(25896d.
210
619
525
940
840
1008
945
630
630

25896
80

83)2071680(24960d.
166
411
332
796
747
498
498
0

12)24960d.
20)2080s.
£104. Ans.

20. As in **19**, I can reduce to pence.

$$\begin{array}{r} 24960 \times \\ 1{\cdot}05 \\ \hline 124800 \\ 24960 \\ \hline 26208{\cdot}00 \end{array}$$

And if I divide £113, 5s. $10\frac{4}{8}$d. reduced to pence by this, I obtain the amount of £1, since the last calculation of interest.

$$26208 \overline{)27190{\cdot}8} \left(1{\cdot}0375 \right.$$

$$\begin{array}{r} 26208 \\ \hline 98280 \\ 78624 \\ \hline 196560 \\ 183456 \\ \hline 131040 \\ 131040 \\ \hline \cdots\cdot \end{array}$$

$\frac{375}{10000} \div \frac{5}{100} = \frac{75}{100}$, or $\frac{3}{4}$, or 9 months. Ans.

21. We must first calculate the amount of the principal 4 years hence.

Since $4\frac{5}{6} = 5 - \frac{1}{6}$, it will be easier to multiply by 5 and subtract $\frac{1}{6}$.

If then we divide the amount given by this amount thus found, we ought to obtain a quotient of 1·04375; the decimal *part of this* ÷ ·05 will *give* ·875, *or* $\frac{7}{8}$ *of a year*, which is the answer given in the book. It is very probable that to simplify numbers, the fractions will have to be considered slightly greater or less.

22. In this case we should proceed as in the last example until we found the 1·04375, and of this we should divide the decimal part by $\frac{7}{8}$, or ·875, which would give us ·05; this would show us that the interest was calculated at 5 per cent. Ans.

23.

$$\begin{array}{r} £4250 \\ 1{\cdot}05 \\ \hline 21250 \\ 4250 \\ \hline £4462{\cdot}50 \\ 1{\cdot}05 \\ \hline 223125 \\ 44625 \\ \hline £4685{\cdot}625 \\ 1{\cdot}025 \\ \hline 23428125 \\ 9371250 \\ 4685625 \\ \hline £4802{\cdot}765625 \\ 20 \\ \hline 15{\cdot}312500\text{s.} \\ 12 \\ \hline 3{\cdot}75000\text{d.} \end{array}$$

Ans. £4802, 15s. $3\frac{3}{4}$d.

24. To find the simple interest.

$$£21\tfrac{1}{2} \times 4 \times \tfrac{5}{100},$$

$$\text{or } £\frac{43 \times 4 \times 5}{2 \times 100},$$

or £4, 6s. 0d.

To find the compound interest.

£21·5
1·05
1075
215
£22·573
1·05
112865
22573
£23·70165
1·05
11850825
2370165
£24·8867325
1·05
1244336625
248867325
26·131069125
21·5
£4·631069125
20
12·6213825
12
7·4565900

Ans. £4, 12s. $7\frac{45659}{100000}$d.

25.

£	s.	d.
5191	13	$7\frac{1}{5}$

20
103833s.
12
1246003·2d.

104)1246003·20(11980
104
206
104
1020
936
840
832
832
832

104)11980800(115200
104
158
104
540
520
208
208

Subtracting present wo from future value,

1246003·2d.
1152000
12)94003·2d.
20)7833s. $7\frac{1}{5}$d.
£391, 13s. $7\frac{1}{5}$d.

Ans. £391, 13s. $7\frac{1}{5}$d.

26. £17·5
1·03

525
175

£18·025
1·03

54075
18025

£18·56575
1·03

5569725
1856575

£19·1227225
17·5

£1·6227225
20

12·45445ȣȣs.
12

5·4534ȣȣd.

C. I. £1, 12s. 5·4534d.
S. I. $£\frac{3\cdot5}{2}\times3\times\frac{3}{100}$,
or £1, 11s. 6d. Ans.

27. Let us reduce the principal to £s.

32)23·

12)2·71875d.

20)12·22656625s.

£134·611328125

And this we must divide by 1·025 three times.

131·328125
1·025)134·611328125
1025

3211
3075

1361
1025

3363
3075

2882
2050

8328
8200

1281
1025

2562
2050

5125
5125

128·125
1·025)131·328125
1025

2882
2050

etc. as before.

1·025)128·125(125
1025
2562
etc.

Ans. £125.

28. £119
1·04
476
119
£123·76
1·04
49504
12376
£128·7104
1·04
5148416
1287104
£133·858816
£119
£14·858816 C. I.
£14·28 S. I.
£·578816 diff.
20
11·57632s.
12
6·91584d.

Ans. 11s. $6\frac{28625}{31250}$d.

29. If P be the principal, the simple interest for 2 years at r per cent. is $\frac{2.P.r}{100}$, and the compound interest is

$$P\left(\frac{100+r}{100}\times\frac{100+r}{100}\right)-P$$
$$=\frac{200Pr+Pr^2}{10000},$$

and this less $\frac{2.P.r}{100}$

$$=\frac{Pr^2}{10000}=\frac{Pr}{100}\times\frac{r}{100},$$

or the interest on the interest for 1 year.

30. We must multiply £260, 5s. by 1·055 three times over, and from the final product subtract £260, 5s. The difference will be the compound interest wanted.

31. Since $3\frac{1}{3}$ will not reduce to a terminating decimal, it is better to do this by Vulgar Fractions, thus—

$$112\tfrac{1}{2}\times\frac{103\frac{1}{3}}{100}\times\frac{103\frac{1}{3}}{100}\times\frac{103\frac{1}{3}}{100}$$
$$=£\frac{225}{2}\times\frac{310}{300}\times\frac{310}{300}\times\frac{310}{300}$$
$$=£\frac{31\times31\times31}{2\times12\times10}$$
$$=£124,\ 2s.\ 7d.;$$

∴ C. I. is £11, 12s. 7d. Ans.

32. All we have to do is to ıltiply £725 four times by ⁵5.

33.

12) 2·16d.

20)13·18s.

£1591·659

101)159165·9(1575·9
101

581
505

766
707

595
505

909
909

£1575·9. Amount at end f 2nd year.

102)157590(1545
102

555
510

459
408

510
510

£1545 is the amount at end of 1st year.

103)154500(1500
103

515
515

Ans. £1500.

34. See Hints, p. 351.

35. Simple interest is

$$£383\tfrac{17}{60} \times 2\tfrac{1}{2} \times \frac{3\frac{3}{4}}{100}$$

$$= £\frac{22997 \times 5 \times 3}{60 \times 2 \times 4 \times 20}$$

$$= £35,\ 18s.\ 7\tfrac{7}{8}d.$$

Compound interest

$$= £383\tfrac{17}{60} \times \tfrac{415}{400} \times \tfrac{415}{400} \times \tfrac{815}{800} - £383\tfrac{17}{60}$$

$$= £37,\ 0s.\ 5\tfrac{1026\,96}{6144000},$$

and the difference is

£1, 1s. $9\frac{36279}{256000}$d. Ans.

36. £1000 × 1·04 × 1·03 × 1·02 × 1·01 will give us the amount.

£1103·55024
£1000

£103·55024

or £103, 11s. 0·0576d. Ans.

37. We must divide the amount by £1000 × 1·04 × 1·03 × 1·01, and we shall get the multiplier for the third year, viz. 1·02; ∴ the rate is 2 per cent. Ans.

38. $3^2 : 6^2 = £10 : £x$.

$x = £\frac{36 \times 10}{9}$, or £40. Ans.

39. See Hints, p. 351.

40. $2^3 : 3^3 = £8 : £x$.

$x = £\frac{27 \times 8}{8}$, or £27. Ans.

41. $5^2 : 8^2$ = area of one : area of other
= 25 : 64. Ans.

42. 25 : 64 = 20 sq. ft. : x ft.

$x = \frac{64 \times 20}{25} = 51\frac{1}{5}$ ft. Ans.

43. $8^3 : 11^3$ = cub. ft. of one cube : cub. ft. of other
= 512 : 1331. Ans.

44. 1331 : 512 = 363 cub. ft. : x cub. ft.

$x = \frac{512 \times 363}{1331} = 139\frac{7}{11}$ c. ft.
Ans.

45. $3^2 : 4^2$ = area of one circle : area of other
= 9 : 16. Ans.

46. 9 : 16 = 27 sq. ft. : x sq. ft.

$x = \frac{16 \times 27}{9} = 48$ sq. ft. Ans.

47. $5^2 : 4^2 = 25 : 16$;
∴ the diameter of the other plate is $\sqrt{25 - 16}$, or 3 ft. Ans.

48. $3^3 : 4^3$ = contents of one sphere : contents of other sphere = 27 : 64. Ans.

49. 27 : 64 = 270 : x;
∴ x = 640 cub. ft. Ans.

50. $\sqrt{64} : \sqrt{100} = 10$ ft. : x ft.

$x = \frac{10 \times 10}{8} = 12\frac{1}{2}$ ft. Ans.

CHAPTER XIX.

1. $\frac{8}{8+5+7}$ of 100 = 40.

$\frac{5}{20}$ of 100 = 25,

$\frac{7}{20}$ of 100 = 35,

40 : 25 : 35. Ans.

2. $\frac{\frac{1}{2}}{\frac{1}{2}+1+1\frac{1}{2}+2}$ of 1000

$= \frac{1}{10}$ of 1000 = 100,

$\frac{1}{5}$ of 1000 = 200,

$\frac{3}{10}$ of 1000 = 300,

$\frac{2}{5}$ of 1000 = 400;

∴ 100, 200, 300, 400. Ans.

3. $\frac{2}{2+112+103}$ of 350

$= 3\frac{7}{31}$

$\frac{112}{217}$ of 350 = $180\frac{20}{31}$,

$\frac{103}{217}$ of 350 = $166\frac{4}{31}$,

$3\frac{7}{31}$, $180\frac{20}{31}$, $166\frac{4}{31}$. Ans.

4. A ought to receive $\frac{1000}{1000+2000+3000}$ of £600, or £100.

B ought to receive $\frac{2000}{6000}$ of £600, or £200.

C ought to receive $\frac{3000}{6000}$ of £600, or £300.

5. A would have $\frac{1000 \times 7}{1000 \times 7 + 2000 \times 4 + 3000 \times 3}$ of £60, or $\frac{7}{24}$ of 600, or £175.

B $\frac{8}{24}$ of 600, or £200.

C $\frac{9}{24}$ of 600, or £225.

6. C receives £300 − £2 × 60 = £180.

£60 : £180 = £1000 : £x.

$x = £\frac{180 \times 1000}{60} = £3000$

Ans.

7. See Hints, p. 352.

8. 12 : 5 = £240 : C's share;

∴ C's share = $£\frac{5 \times 240}{12}$

= £100;

∴ A's share was £200.

£240 : 200 = 12 : A's time.

A's time = $\frac{12 \times 200}{240}$ = 10 mo.

Ans.

9. Let us take as our unit the value of use of £1 for 1 day. A had £100 in 100 days, or 10000 units; B had £200 in 60 days, or 12000 units; and C had £300 in 40

days, or 12000 units; ∴ A had $\frac{10000}{10000+12000+12000}$, or $\frac{10}{34}$ of £680, or £200. B and C have $\frac{12}{34}$ of £680, or £240 each. Ans.

10. £240 : £200 = 12000 : number of units A received; ∴ A received 10000 units; and this ÷ £100 will give us 100 days. Ans.

11. £200 : £240 = 10000 : number of units B received; ∴ B received $\frac{240 \times 10000}{200}$, or 12000 units; and this ÷ 60 gives £200 on B's capital. Ans.

12.

3725·39
·979

20)3647·15681 value of 20 mks.

£182·3578405
20

7·15681 shs.
12

1·88172 d.

which is very nearly £182, 7s. 2d. Ans.

13.

1 ml.
1760

1760 yds.
3

5280 ft.
12

39·3708)63360 in.(1609
393708

2398920
2362248

3667200
3543372

remainder not asked for.

Ans. 1609 metres.

14. This is simply to divide 24 into two parts which are in the ratio of 8·5 : 5·1, or 5 : 3; ∴ the fire is lighted $\frac{3}{5+3}$ of 24, or 9 hrs. Ans.

15. $\frac{1000}{50} = 20$; ∴ I pay 10s. 2½d. × 20, or £10, 4s. 2d.; whereas 2½d. × 10000 = £10, 8s. 4d. I therefore gain 50 pence, or 20 piastres.

16. $\frac{1078}{98} = 11$. I ∴ pay £11; but 2½d. × 1078 = £11 4s. 7d. I therefore gain 4s. 7d.

17. 2000 sovereigns is worth 2000×98 piastres, and a 50-piastre piece is worth 49 piastres; and $\frac{2000 \times 98}{49} = 4000$;
∴ I ought to receive
4000 50-piastre pieces. Ans.

18. C has $\frac{4}{2 + 3\frac{1}{2} + 4 + 5}$ of £1450 in the business, or $\frac{8}{29}$ of £1450, or £400, and he has it in for 5 units of time; ∴ he receives 2000 units, whereas A has $\frac{4}{29}$ of £1450 for 3 units of time; he ∴ receives 600 units.

units. units.
2000 : 600 = £200 : £x.
£x = £60. Ans.

Note.—Though the amount of the profit to be divided is given, it is not required to solve the problem.

19. Since we know A's share of the profits and the differences, we can at once tell the others' profits to be £140, £200, and £300. Since A had his money in the business 3 months, and C 5 months, we can at once compare A's and C's money by the profits they received.

A for his money for one month received £20, and C for his money for one month received £40; ∴ A's money : C's money = 1 : 2; but the *difference between* them is £200; ∴ A had £200, and C £400. Therefore B had £350, and D £500 in the business respectively. C had 400×5 share of the profits, and D must have had

$400 \times 5 \times \frac{300}{200}$, or 3000 shares;

but since D had £500 in the business, it must have been in 6 months. Similarly it can be shown B's money was in 4 months. Ans.

20. B receives $\frac{1}{4}$ (210 − 30), or £90, and A £120.

A's money for 1 month earns £30. B's money for 1 month earns £18;
∴ A's money : B's money = 5 : 3.
We have to find a fraction = $\frac{5}{3}$ whose terms differ by 200.

$$\frac{5}{3} = \frac{3+2}{3} = \frac{3 \times 100 + 200}{3 \times 100};$$

∴ A had £500, and B £300. Ans.

21.

s.	*d.*	£	*s.*	*d.*
19	7)	35	6	5
12	)	20		
235d.		706		
		12		
		8477 (	36	
		705		
		1427		
		1410		
		17		

and 17d. $= \frac{1700}{12}$ pfennings, or $141\frac{2}{3}$ pfennings.

Ans. 36 20-mark pieces and $141\frac{2}{3}$ pfennings.

22. Our table is this:

10 mace make 1 tael.
$\frac{1}{80}$ tael „ 1 penny.
54 pence „ 1 C. dollar.

100 C. dollars
54
5400 pence
$\frac{1}{80}$
$\frac{540}{8}$ tael.
$= 67$ taels 5 mace. Ans.

23. $\frac{2\frac{1}{2}\text{d.} \times 1900}{9\frac{1}{2}\text{d.}}$, or 500 lire.

24. $\frac{1\text{s. } 1\frac{1}{4}\text{d.} \times 100}{1\text{s. } 8\text{d.}}$, or $66\frac{1}{4}$ H. florins. Ans.

25. The real debt is $£\frac{1000}{25}$, or £40. $9\frac{1}{2} \times 1000\text{d.} = £39$, 11s. 8d.

Loss, 8s. 4d. Ans.

26. $\frac{530\text{s.}}{1\text{s. } 1\frac{1}{4}\text{d.}} = \frac{530 \times 12 \times 4}{53}$
$= 480$. Ans.

27. 1 mark + 1 S. kronor = 2s. $1\frac{1}{4}$d.

$$\frac{1010 \times 12 \times 4}{101} = 480.$$

Ans. 480 of each.

28. $\frac{£78,\ 16\text{s. } 2\text{d.}}{£1 + 1\text{s.} + 4\frac{1}{2}\text{s.} + £\frac{1}{3}}$

$$= \frac{1576\frac{1}{6}\text{s.}}{32\frac{1}{6}\text{s.}} = \frac{9457}{193} = 49.$$

Ans. 49 of each.

29. $\frac{108 \times 20 \times 25}{100}$ dollars, or 500 dollars.

$$\frac{£100}{4\text{s. } 2\text{d.}} = \frac{100 \times 20}{4\frac{1}{6}}$$

$= \frac{100 \times 20 \times 6}{25}$ dollars, or 480 dollars; ∴ I gain 20 dollars. Ans.

30. $\frac{4\text{s. } 2\text{d.} \times 55\frac{1}{10}}{9\frac{1}{2}\text{d.}}$,

or $\frac{50 \times 551 \times 2}{10 \times 19}$ drachmas,

or 290 drachmas. Ans.

31. Since an Am. dollar = 4s. 2d.

$\$1 = 4\frac{1}{6}$s.
$\$2 = 4\frac{1}{6}$s. × 2;

hence the constant multiplier is $4\frac{1}{6}$. Ans.

$$\frac{\text{Am. dollar}}{\text{Sw. kronor}} = \frac{4\text{s. } 2\text{d.}}{1\text{s. } 1\frac{1}{4}\text{d.}} = \frac{50 \times 4}{53};$$

∴ the constant multiplier is $\frac{50 \times 4}{53}$, or $3\frac{41}{53}$. Ans.

32. To form our table—

$\frac{4}{7}$ goose = 1 fowl.
$\frac{14}{3}$ fowls = 1 pig.
$\frac{11}{4}$ pig = 1 ox.
$\frac{5}{3}$ oxen = 1 horse;

$\therefore$ 9 horses $= 9 \times \frac{5}{3} \times \frac{11}{4} \times \frac{14}{3} \times \frac{4}{7}$ geese $=$ 110 geese. Ans.

33. 110 geese are worth $\frac{7 \times 110}{4}$ fowls.

$\frac{7 \times 110}{4}$ fowls are worth $\frac{7 \times 110 \times 3}{4 \times 14}$ pigs,

and 9 horses are $= \frac{5 \times 9}{3}$ oxen,

but $\frac{7 \times 110 \times 3}{4 \times 14}$ pigs $= \frac{5 \times 9}{3}$ oxen.

$\frac{\text{pig}}{\text{ox}} = \frac{5 \times 9 \times 4 \times 14}{3 \times 7 \times 110 \times 3} = \frac{4}{11}$,

or 11 pigs are worth 4 oxen.

34.

horses	oxen	goose
4	2	1
$\frac{5}{3}$		
$(4 \times \frac{5}{3} + 2)8\frac{2}{3}$ oxen		
$\frac{11}{4}$		
$23\frac{5}{6}$ pigs		
$\frac{14}{3}$		
$111\frac{2}{3}$ fowls		
$\frac{4}{7}$		

(adding 1) $64\frac{5}{9}$ geese. Ans.

35.

$63\frac{5}{9}$ geese $= \frac{572 \times 7}{9 \times 4}$ fowls.

4 horses $= 4 \times \frac{5}{3}$ oxen, and these $+ 2 = 8\frac{2}{3}$ oxen.

$\frac{26}{3}$ oxen $= \frac{26 \times 11}{3 \times 4}$ pigs, but

$\frac{26 \times 11}{3 \times 4}$ pigs $= \frac{572 \times 7}{9 \times 4}$ fowls.

$\frac{\text{pig}}{\text{fowl}} = \frac{572 \times 7 \times 3 \times 4}{9 \times 4 \times 26 \times 11} = \frac{14}{3}$;

$\therefore$ 3 pigs are worth 14 fowls. Ans.

36. $\frac{43 + 39 + 37 + 51 + 50}{5}$,

or 44. Ans.

37. $44 \times 5 - (43 + 37 + 51 + 50) = 220 - 181 = 39$. Ans.

38. I purchase $(11 + 12 + 13)$ eggs for $(9 + 8 + 7)$d., or 36 eggs for 24d., or 3 eggs for 2d. Ans.

39. If for 3 eggs I give 2d., and for one $\frac{9}{11}$d., and another $\frac{2}{3}$d.; $\therefore$ for the third I give $(2 - \frac{9}{11} - \frac{2}{3})$d.,

or $\frac{66 - 27 - 22}{33}$d., or $\frac{17}{33}$d. Ans.

40. The eggs cost me $5 \times 8 + 7 \times 6 + 9 \times 4$ units of price, and this $\div$ by $5 + 7 + 9$ will

give us the average price at which I buy and sell.

$$\frac{40+42+36}{21} = 5\tfrac{13}{21},$$

or I sell them per dozen at $5\frac{13}{21}$ of the unit of price. Ans.

41. We do not require to know the number of eggs to find the cost. The cost of the second parcel must be $\frac{7}{5}$ of $\frac{6}{8}$ of £15, 15s.,
or £16, 10s. 9d. Ans.

And that of the third parcel, $\frac{2}{5}$ of $\frac{4}{8}$ of £15, 15s.,
or £14, 3s. 6d. Ans.

42. I have to divide £46, 9s. 3d. into the proportion of

$$5\times 8 : 7\times 6 : 9\times 4.$$

$$\frac{5\times 8}{40+42+36} \text{ of } \frac{929\frac{1}{4}}{20}$$

$$= £\frac{40\times 3717}{118\times 20\times 4}$$

$$= £15, 15s. \text{ Ans.}$$

Similarly the second parcel cost $£\frac{42\times 3717}{118\times 20\times 4}$,
or £16, 10s. 9d. Ans.

And the third parcel,

$$£\frac{36\times 3717}{118\times 20\times 4},$$

or £14, 3s. 6d. Ans.

43. The 4 summer months contain 123 days, and the 4 winter months 120 days; ∴ it loses altogether $120\times\frac{1}{4}$ sec., and $3\times 1\frac{1}{2}$ sec., or altogether $34\frac{1}{2}$ sec., and this ÷ gives

$$\frac{69}{243\times 2}, \text{ or } \frac{23}{162} \text{ sec. a day. Ans.}$$

If the clock is supposed neither to win nor lose in the spring and autumn, we must divide $34\frac{1}{2}$ sec. by 365.

44. The entire rent is $25\frac{1}{3}$s. $\times$ 250 + $21\frac{2}{3}$s. $\times$ $62\frac{1}{2}$ + 60s. $\times$ $19\frac{1\frac{1}{2}}{4}$, or £442, 10s., and there are altogether $331\frac{7}{8}$ acres, and $£442\frac{1}{2} \div 331\frac{7}{8}$

$$= £\frac{885\times 8}{2\times 2655}$$

$$= £1, 6s. 8d. \text{ Ans.}$$

45. Since there are $331\frac{7}{8}$ acres altogether, and the average price is $£1\frac{1}{3}$, the entire rental is $£442\frac{1}{2}$; but the rents of the first 2 portions are £316, 13s. 4d., and £67, 14s. 2d.; ∴ the rent of the third is £58, 2s. 6d., and this divided by $19\frac{3}{8}$ will give us £3. Ans.

46. Since these two twos cancel one another, all we have to do is to divide 45 by 3, and add on 2 for one, and subtract 2 for the other.

Ans. 13, 15, 17.

47. The women earn $\frac{16}{15+16+15}$ of £460, or £160. Ans.

48. If there are 48 women there are 60 children, and all the children earn $\frac{15}{46}$ of £460, or £150; and $\frac{£150}{60}$ = £2, 10s., the earning of each child. Ans.

49. $\frac{6\times3\times1}{60+40+18}$ of £2360, or £360. Ans.

50. There are $\frac{4}{6}$ of 1080 men, or 720 men, and they earned $\frac{60}{118}$ of £2360; and ∴ each man earned

$$£\frac{60\times2360}{118\times720}, \text{ or } £\frac{5}{9};$$

∴ his weekly wages were $£\frac{5}{9}$, or 11s. $1\frac{1}{3}$d. Ans.

CHAPTER XX.

1. See Hints, p. 354.

2. £28 × 20, or £560. Ans.

3. I receive £19 × 280, and with this money I buy shares at £28; ∴ I buy $\frac{19\times280}{28}$, or 190 shares. Ans.

4, 5. See Hints, p. 354.

6.

$$£12002\frac{12\,4\frac{1}{2}}{20}\times\tfrac{3}{9\,3},$$

$$\text{or } £12002\frac{99}{8\times20}\times\tfrac{3}{9\,3},$$

$$\text{or } £\frac{1920419}{160}\times\tfrac{1}{3\,1},$$

or £387, 3s. $7\frac{1}{2}$d. Ans.

7. $\frac{£387,\ 3s.\ 7\frac{1}{2}d.}{£3}$ will give the number of cents. as

$$129\tfrac{29}{480},$$

and £12002, 12s. $4\frac{1}{2}$d. ÷ by $129\frac{29}{480}$ gives 93 as price of stock. Ans.

8. $4\frac{1}{2} : 6 = 100 : x.$

$$x=\frac{6\times100\times2}{9}$$

$$=133\tfrac{1}{3}. \text{ Ans.}$$

9. $133\frac{1}{3} : 100 = 6 : x.$

$$x=\frac{100\times6\times3}{400}$$

$$=4\tfrac{1}{2}. \text{ Ans.}$$

10. My old income is

$$£\frac{5000}{100} \times 4, \text{ or } £200.$$

I receive $\frac{5000}{100} \times 108$, and with this I buy $\frac{5000}{100} \times 108 \div 120$ cents., and my new income is this $\times 5$,

or $£\frac{5000 \times 108 \times 5}{100 \times 120}$, or $£225$;

$\therefore$ my income is increased by $\underline{£25}$. Ans.

11. As this question stands, it is not a converse problem to **10.**

My old income was $£200$;

$\therefore$ my new $£225$.

$\frac{225}{5} = 45$ cents., and these cost me 45×108, and 45×108 contains 50 (the number of cents. I had), $97\frac{1}{2}$ times. Price, $\underline{97\frac{1}{2}}$. Ans.

12.

$$£\frac{329 \times 104}{3\frac{1}{2}}$$

$$= \underline{£9776}. \text{ Ans.}$$

13. I have $\frac{9776}{104}$ cents., and $329 \div \frac{9776}{104}$ will give me the income derived from each cent.

$\frac{329 \times 104}{9776} = \frac{7}{2}$, or $\underline{3\frac{1}{2}}$. Ans.

14.

$$\frac{£340,\ 4\text{s. } 8\text{d.} \times 63\frac{1}{2}}{4}$$

$$= £\frac{20414 \times 127}{20 \times 3 \times 4 \times 2}$$

$$= \underline{£5401,\ 4\text{s. } 1\text{d.}} \text{ Ans.}$$

15. See Hints, p. 354.

$$\frac{£340,\ 4\text{s. } 8\text{d.} \times 63\frac{5}{8}}{4}$$

$$= £\frac{20414 \times 509}{20 \times 3 \times 4 \times 8}$$

$$= \underline{£5411,\ 16\text{s. } 8\frac{3}{4}\text{d.}} \text{ Ans.}$$

16. My net income for each cent. is $\frac{235}{240}$ of $£3$. This proportion then will give us the answer.

$£3 : £\frac{235}{240}$ of $3 = 100 : x$.

$\frac{235 \times 3 \times 100}{240 \times 3}$, or $\underline{97\frac{11}{12}}$. Ans.

17. I spend in brokerage $\frac{100}{3}$, or $£33\frac{1}{3}$, and $33\frac{1}{3} \div \frac{1}{8} = 266\frac{2}{3}$ is the number of cents. I buy, or $\underline{£26666\frac{2}{3}}$. Ans.

18. I. $3\frac{1}{3} : 3 = 100 : x$.

$$x = \frac{3 \times 100 \times 3}{10}$$

$$= \underline{90}. \text{ Ans.}$$

19. $£\frac{204}{3} \times 90$,

or $\underline{£6120}$. Ans.

20. $(103\frac{7}{8} - 97\frac{4}{8}) \times \frac{1950}{100}$,

or $£6\frac{3}{8} \times \frac{39}{2}$,

or $\underline{£124,\ 6\text{s. } 3\text{d.}}$ Ans.

21. $104 - \frac{1}{8} = 103\frac{7}{8}$,

$97\frac{3}{8} + \frac{1}{8} = 97\frac{4}{8}$,

$103\frac{7}{8} - 97\frac{4}{8} = 6\frac{3}{8}$,

and £124, 6s. 3d. contains $6\frac{3}{8}$ $\frac{1989 \times 8}{16 \times 51}$, or $\frac{39}{2}$ times;
∴ money invested was
£1950. Ans.

22. See Hints, p. 354.

23. He made £1000 × $1\frac{3}{8}$, or £1375, of which he had to pay his broker £2000 × $\frac{1}{8}$, or £250.

24. In the one case he receives $\frac{5}{117\frac{1}{2}}$ of the money invested, and in the other $\frac{3}{92\frac{1}{4}}$ of it. And $\frac{10}{235} - \frac{12}{369}$
$= \frac{2}{47} - \frac{4}{123}$
$= \frac{246 - 188}{47 \times 123} = \frac{58}{47 \times 123}$,
and since $\frac{58}{47 \times 123}$ of the sum = £29, the sum is
$\frac{47 \times 123 \times 29}{58}$,
or £2890, 10s. Ans.

25. The old income : new income = $3 : \frac{99 \times 4}{110}$
$= 5 : 6$;
∴ $\frac{1}{6}$ of the new income
= £6, 4s.;
∴ his original income is £31, and £31 : £3 = $1012\frac{2}{3} : x$,
or $x = \frac{3038 \times 3}{3 \times 31} = 98$. Ans.

26. $\frac{3}{64\frac{1}{4}} \sim \frac{5}{102\frac{3}{4}}$
$= \frac{12}{257} \sim \frac{20}{411}$
$= 4\left\{\frac{1233 \sim 1285}{257 \times 411}\right\}$
∴ the United States investment is better by $\frac{208}{257 \times 411}$, or $\frac{208}{105627}$, and this on £1000 will give a difference of $\frac{208000}{105627}$, or £1, 19s. $4\frac{213352}{385209}$d. Ans.

27. $\frac{5}{105}, \frac{4}{96}, \frac{3}{90}$,
or $\frac{1}{21}, \frac{1}{24}, \frac{1}{30}$,
or $\frac{40, 35, 28}{3 \cdot 7 \cdot 8 \cdot 5}$. Ans.

28. $3600 : 100 = 3519 : x$.
$x = \frac{100 \times 3519}{3600}$
$= 97\frac{7}{8}$. Ans.

The income is £$3\frac{1}{2} \times 36$, or £126. Ans.

29. $\frac{126}{3\frac{1}{2}} \times 97\frac{7}{8}$
$= \frac{126 \times 2 \times 391}{7 \times 4}$
= £3519. Ans.

30. His old income was $\frac{19200}{100} \times 3$, or £576.
His new income is
£$\frac{19200 \times 85 \times 4}{100 \times 96}$,
or £680. Ans.

31. See Hints, p. 355.

32. If £295 is the net income after 4d. in the £ is paid, the gross income is $\frac{240}{236}$ of 295, or $\frac{60}{59}$ of 295, or £300.

$$\frac{300 \times 87}{3} = \underline{£8700}. \text{ Ans.}$$

33. We must know the price of the stock.

$$£\frac{8700 \times 3}{87} = £300.$$

$$\frac{300}{295} = \frac{300 \times \frac{240}{300}}{295 \times \frac{240}{300}}$$

$$= \tfrac{240}{236};$$

$\therefore$ $240 - 236 = 4$d.;

$\therefore$ tax is $\underline{4\text{d. in } £1}$. Ans.

34. See Hints, p. 355.

35. $\frac{5}{110} + \frac{4}{100} + \frac{5}{110}$.

$$\frac{50 + 44 + 50}{11 \times 10 \times 10} = \tfrac{144}{1100};$$

$\therefore$ $£\frac{144}{1100}$ is the interest on every £3.

$$2309\tfrac{2}{11} \div \tfrac{144}{1100}$$

$$= \frac{254016 \times 1100}{11 \times 144},$$

and this $\times 3$ will be the amount invested, or $\underline{£529200}$. Ans.

36. The gross income is $\frac{240}{236}$ of £470, or £480.

$£\frac{480 \times 90\frac{3}{4}}{3}$ is the sum invested, or $\underline{£14520}$. Ans.

37. See Hints, p. 355.

38. $\frac{3}{96} - \frac{3}{97} = \frac{3}{96 \times 97}$,

or $\frac{1}{32 \times 97}$;

$\therefore$ the money invested is $£32 \times 97$, or $\underline{£3104}$. Ans.

39. See Hints, p. 355.

40. He receives as dividend $\frac{7440 \times 1\frac{1}{2}}{93}$, or £120.

He loses by the transfer $1\frac{1}{8}$ per cent., or $83\frac{7}{10}$; $\therefore$ altogether in 3 months he makes $£36\frac{3}{10}$, or in 1 year $£145\frac{2}{10}$, and this on £7440 is $\underline{1\frac{177}{186} \text{ per cent.}}$ Ans.

41. See Hints, p. 355.

42. $\frac{240}{4 \times 3} = 20$ cents.

He receives $£20 \times 87\frac{1}{4}$, or £1745.

$\frac{£1745}{£174\frac{1}{2}} = 10$, the shares.

His new income on this portion of his money is £60

$+£40$, and if he receives $£100$ for 10 cents., he receives $£10$ per cent. Ans.

43. The true value of $£8820$ when he sells is $1\frac{1}{45}$ of $£8820$, or $£46 \times 196$; $\therefore$ the gain on each $£$ is $£\frac{25}{46 \times 196}$; but the income value of each $£$ in the second case is $\frac{4}{98}$, and $\frac{4}{98} - \frac{25}{46 \times 196}$ $= \frac{368 - 25}{2 \times 49 \times 23 \times 2 \times 2} = \frac{343}{92 \times 98}$ $= \frac{3\frac{1}{2} \times 98}{92 \times 98} = \frac{3\frac{1}{2}}{92}$; $\therefore$ the value of each cent. when he sold out was 92, and $\frac{5}{40}$ of this is 90. Ans.

44. The income in the 1st case in each $£1$ is

$$£\frac{3\frac{1}{4}}{93\frac{3}{8}}, \text{ or } £\frac{26}{747},$$

and that in the 2nd case is

$$£\frac{4\frac{1}{4}}{105\frac{3}{8}}, \text{ or } \frac{34}{843},$$

$$\text{and } \frac{34}{843} - \frac{26}{747} = \frac{8466 - 7306}{3 \times 281 \times 3 \times 83}, \text{ or } \frac{1160}{3 \times 281 \times 3 \times 83},$$

and as often as this is contained in $\frac{34 \times 1595}{843}$, so many pounds were invested, and the income was $\frac{26}{747}$ of this sum, or

$$£\frac{34 \times 1595 \times 3 \times 281 \times 3 \times 83 \times 26}{843 \times 1160 \times 747},$$

that is, $£\frac{17 \times 11 \times 13}{2 \times 3}$, or $£405$, 3s. 4d. Ans.

46. His original income was $£\frac{10257\frac{1}{2} \times 3}{100}$, or $£\frac{61545}{200}$; $\therefore$ his new income would have been

$$£\frac{61545}{200} + \frac{2180}{200}, \text{ or } £\frac{2549}{8}.$$

And $£\frac{2549}{8} \div 3\frac{1}{4} \times 93\frac{1}{4}$ will give us what he invested; and $\therefore$ what he received for his stock, and this is $£\frac{2549}{8} \times \frac{4}{13} \times \frac{373}{4}$, and this $\div 102\ \frac{57\frac{1}{2}}{100}$ will give us the selling price of his stock; or $£\frac{2549}{8} \times \frac{4}{13} \times \frac{373}{4} \times \frac{200}{20515}$, or $£\frac{127445}{143}$ or $89\frac{18}{143}$ calling this $89\frac{1}{8}$. He sold out at $89\frac{3}{8}$, and received $£\frac{205155}{800}$

$\times \frac{715}{8}$, and with this he obtained a new income $£\frac{205515}{200} \times \frac{715}{8} \times \frac{13}{2 \times 187}$ increasing the 1st fraction by $£\frac{55}{200}$, or 27s. 6d. to make it divisible by 5×187 the product becomes $\frac{204499}{64}$, or £319, 7s. $2\frac{1}{2}$d. But the old income was $£\frac{10257\frac{1}{2} \times 3}{100}$ or $£\frac{61545}{200}$, or £207, 14s. 6d.; ∴ the improvement is £11, 12s. $8\frac{1}{2}$d.

Ans.

47. Let the former stock be at par, then for the other I receive as income $4\frac{12}{17}$ per cent.

$4 : 4\frac{12}{17} = 100 : x$

$x = \frac{801 \times 100}{17 \times 4}$

$= 119\frac{11}{17}$ price of 4 per cents.

5 p. c. : 4 p. c. = 100 : $119\frac{11}{17}$

= 1700 : 2000

= 17 : 20.

Ans.

48. $\frac{5}{127} \sim \frac{5\frac{1}{2}}{135}$

$= \frac{5 \times 2 \times 135 \sim 11 \times 127}{127 \times 2 \times 135}$,

or $\frac{47}{127 \times 270}$; and as often as this is contained in $£4\frac{14}{20}$, so many £s are there invested.

$£\frac{94 \times 127 \times 276}{20 \times 47}$, or £3429.

Ans.

49. $£3429 \times \frac{5}{127}$, or £135,

and $£3429 \times \frac{5\frac{1}{2}}{135}$,

or £139, 14s. Ans.

50. Eggs at 8d. a dozen cost $\frac{8}{12}$d. each, but if sold at 7s. 6d. a 100, are sold at $\frac{90}{100}$ each.

$\frac{9}{10} - \frac{2}{3} = \frac{27 - 20}{10 \times 3}$, or $\frac{7}{30}$.

$\frac{8}{12} : 100 = \frac{7}{30} : x$

$x = \frac{100 \times 7 \times 3}{2 \times 30}$

$= 35$ per cent.

100 eggs cost me $\frac{200}{3}$d., and for this I receive 7s. 6d., and 7s. 6d. – 6s. $6\frac{2}{3}$d. = 1s. $11\frac{1}{3}$d.; and this contains $\frac{2}{3}$d. 35 times; ∴ for 100 eggs I should receive the buying price of 135 eggs, and the answer is the same as before, viz. 35 p. c.

Ans.

MISCELLANEOUS ANSWERS ON PART II.

I.

1. $\frac{5}{6} = \frac{5}{5+1} = \frac{5\times 3}{5\times 3+3} = \frac{15}{18}$. Ans.

2. $\frac{1}{1+4}$ of $10 = 2$,

and $\frac{4}{1+4}$ of $10 = 8$;

$\therefore$ ratio is 2 : 8. Ans.

3. s. s. lbs. lbs.

$36 : 30 = 4 : x$

$x = \frac{30\times 4}{36}$, or $3\frac{1}{3}$ lbs. Ans.

4. Price varies directly as the price of the loaf; $\therefore$ 30s. = some multiplier $\times$ 5; $\therefore$ the multiplier $= \frac{5}{30}$, or $\frac{1}{6}$. Ans.

5. $5\text{d.} = 30 \times \frac{1}{6}\text{d.}$; $\therefore$ the unit must be $\frac{1}{6}$ penny. Ans.

6. We have to find a fraction $= \frac{3}{7}$ whose terms differ by 12.

$\frac{3}{7} = \frac{3}{3+4} = \frac{9}{9+12} = \frac{9}{21}$.

7.
$$17\overline{)1000}\,(58$$
$$85$$
$$150$$
$$136$$
$$14$$

We must then add on $17 - 14 + 1$ or 4. Ans. 1004, into which 17 will go 59 times + 1. Ans.

8.
$$19{\cdot}01$$
$$20{\cdot}3$$
$$19{\cdot}07$$
$$18{\cdot}62$$
$$4)77{\cdot}00$$
$$19{\cdot}25.$$ Ans.

$\frac{1000}{20\cdot 3}$ of $812 = 40000$. Ans.

II.

9. $\frac{2}{2+3+4+5}$ of $98 = 14$.

$\frac{3}{14}$ of $98 = 21$.

$\frac{4}{14}$ of $98 = 28$.

$\frac{5}{14}$ of $98 = 35$.

If the 3 represent 630, since $630 = 21 \times 30$, the unit is 30, and the 5 represents 35×30, or 1650. Ans.

10. $\frac{60\times 1760\times 3\times 2}{60\times 60\times 4}$, or 44 half feet per quarter second. Ans.

11. Since one man is running 6 miles an hour in the opposite direction to the train, which is going 56 miles an hour, he is

travelling at 56 – 6, or 50 miles an hour. The other man is travelling at the rate of 42 + 8, or 50 miles an hour; ∴ they separate one from another at the rate of 50 + 50 miles an hour.

sec. sec. mls. mls.
$60 \times 60 : 36 = 100 : x$

$x = \frac{36 \times 100}{60 \times 60}$, or 1 mile. Ans.

12. $\frac{3}{3+5}$ of $37 = \frac{111}{8} = 13\frac{7}{8}$.
$\frac{5}{8}$ of $37 = \frac{185}{8} = 23\frac{1}{8}$.
Ans.

13. $1 \times 20 \times 12 \times 4 - 112 \times 8$, or $960 - 896$, or 64. Ans.

14. See Hints, p. 356.

15. The L. C. M. of 18 ft. 4 in. and 4 ft. 2 in., or of $18\frac{1}{3}$ ft. and $4\frac{1}{6}$ ft., or of $\frac{110}{6}$ and $\frac{25}{6}$, is $\frac{5.2.11.5}{6}$ ft., and 5 miles contains this $\frac{5 \times 1760 \times 3 \times 6}{5 \times 2 \times 11 \times 5}$, or 288 times. Ans.

16. $\frac{5 \text{ miles}}{288}$ is the L. C. M. of 4 ft. 2 in., and the circumference of the larger wheel.
$\frac{5 \text{ miles}}{288} = \frac{5 \times 1760 \times 3 \times 12 \text{ in.}}{12 \times 3 \times 4 \times 2}$,
or 1100 in.

Now since 1100 is $= 5 \times 2 \times 5 \times 2 \times 11$, and $50 = 5 \times 2 \times 5$, the other wheel may be any measure of 1100, which is also a multiple of 11×2, as $11 \times 2 \times 5$, or $11 \times 2 \times 5 \times 2$, or $11 \times 2 \times 5 \times 5$, etc.

III.

17. $7 \times 9 \times 5 + 2 = 315 + 2 = 317$. Ans.

18.

	614	666	
	3	3	
47	611	663	1
	52	611	
	91	52	
	91	13	

Ans. 13.

19. $\frac{1\text{d.} \times 4 \times 120}{100} = 4\frac{4}{5}\text{d.}$
Ans.

20. $4\frac{1}{5\frac{1}{6}} - 2 = 2\frac{1}{5\frac{1}{6}}$
$= 2\frac{6}{31} = \frac{68}{31}$,
and $\frac{68}{31} \div \frac{8}{7} = \frac{68}{31} \times \frac{7}{8} = \frac{476}{248}$.
Ans.

21. $\frac{\frac{1}{2}}{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}}$ of 317
$= \frac{4}{4+2+1}$ of $317 = 181\frac{1}{7}$.
$\frac{2}{7}$ of $317 = 90\frac{4}{7}$, and $\frac{1}{7}$ of $317 = 45\frac{2}{7}$.

22. The interest in 1st case is $\frac{5}{100}$ of P.
The interest in 2nd case is $\frac{4}{100}$ of P.
The interest in 3rd case is $\frac{3}{100}$ of P.
If $\frac{5}{100}$ of 1st sum $= \frac{4}{100}$ of 2nd sum, one sum $= \frac{4}{5}$ of 2nd sum. Similarly, 2nd sum $= \frac{3}{4}$ of 3rd sum, or 3rd sum is $\frac{4}{3}$ of 2nd sum; ∴ the sums are
$4 : 5 : \frac{4}{3}$ of 5,
or 12 : 15 : 20. Ans.

23. The interests are—
$\frac{5}{100}$P, $\frac{4}{100}$P, $\frac{3}{100}$P,
or 5 : 4 : 3. Ans.

24. If $\frac{5}{100}$ of the sum $= £120$, the sum $= £2400$. Ans.

IV.

25.
$$£50 : £100 = £1 : £x$$
$$6 \text{ mo.} : 1 \text{ mo.}$$
$$x = £\frac{100 \times 1 \times 1}{50 \times 6} = £\frac{1}{3},$$
or 6s. 8d. for £100. Ans.

26. $\frac{5}{100} - \frac{4}{100} = \frac{1}{100}$.
If $\frac{1}{100}$ of amount $= £3$, the amount is £300. Ans.

27.
$$£100 : £1$$
$$300 \text{ days} : 1 \text{ day} = £5 : £x$$
$$x = £\frac{5}{100 \times 300} = £\frac{1}{6000},$$
or $\frac{1}{20}$d. Ans.

28. See Hints, p. 357.

29. We must first subtract the 100s. or £5 from £109, and then divide this by
$$\frac{5+5+3}{3}, \text{ or } \tfrac{13}{5}.$$
$$\frac{104 \times 5}{13} = £40;$$
∴ men have £45,
women ,, £40,
children ,, £24.
Each man has $\frac{45}{100}$, or 9s.
,, woman ,, $\frac{40}{100}$, or 8s.
,, child ,, $\frac{24}{100}$, or $4\frac{4}{5}$d.

30. D has $\frac{23}{14}$ of $\frac{7}{8}$ of $\frac{5}{2}$ of A, or $\frac{23}{4}$ of A. Ans.

31. A has $\frac{2}{5}$ of 100, or £40. Ans.

32. A has $\frac{7}{8}$ of B;
∴ B has $\frac{8}{7}$ of A,
or $\frac{2}{7}$ less than B. Ans.

V.

33. If 1s. 3d. is the interest on the discount at 5 p. c. for 3 years, the discount is
$$\frac{100 \times \frac{5}{4}}{5 \times 3} \text{s., or } \tfrac{100}{12}\text{s.}$$
and the principal is $\frac{115}{15}$ of $\frac{100}{12}$s., or £3, 3s. $10\frac{2}{3}$d. Ans.

34. $\frac{5}{95} - \frac{4}{80} = \frac{80 - 76}{5 \times 19 \times 16}$
$= \frac{1}{380}$;
∴ the sum $= £2\frac{1}{2} \times 380$, or £950. Ans.

35. $\frac{3}{5} = \frac{3}{3+2} = \frac{3\times 12}{36+24} = \frac{36}{60}$. Ans.

36. The numerator $= \frac{4}{4+7}$ of 33, or 12; the denominator $= \frac{7}{4+7}$ of 33, or 21. Ans. $\frac{12}{21}$.

37. Let our unit be $\frac{1}{3.2.2.2.5}$ of the work to be done;

∴ A + B do 20 units each day,
B + C do 15 „ „
C + A do 12 „ „

∴ A + B + C do $\frac{20+15+12}{2}$ units each day, or $\frac{47}{2}$ units;

∴ C does $\frac{7}{2}$ units each day,
and A „ $\frac{17}{2}$ „ „
and B „ $\frac{23}{2}$ „ „

∴ A would take $\frac{3.2.2.2.5.2}{17}$, or $14\frac{2}{17}$ days.

B $\frac{3.2.2.2.5.2}{23}$, or $10\frac{10}{23}$ days.

C $\frac{3.2.2.2.5.2}{7}$, or $34\frac{2}{7}$ days.

38. A's play is $\frac{5}{4}$ of B's, B is $\frac{10}{7}$ of C's; ∴ A is $\frac{5}{4}$ of $\frac{10}{7}$ of C $= \frac{50}{28} = \frac{50\times 20}{28\times 20} = \frac{1000}{560}$;

∴ A ought to give C 440 points. Ans.

39. $\frac{125}{23}$ *It. lire* = 1 R. scudi.
$\frac{2001}{12500}$ *R. scudi* = 1 A. z.

50 A. z. $= \frac{50\times 2001\times 125}{12500\times 23}$
$= 43\frac{1}{2}$ Italian lire. Ans.

40. His gross income is $\frac{240}{235}$ of £705, or £720. Ans.

VI.

41. 4 : 3 = 100 : x.
$x = 75$. Ans.

42. See Hints, p. 357.

43. Let us call the attendance of children on Monday 100. Then the attendance each day is represented by these numbers—

100, 92, 105, 105, 95;

∴ the daily average attendance to this unit is $99\frac{2}{5}$; but 105 − 92, or 13 represents 26 children; ∴ the unit is 2 children, and the daily average attendance, $198\frac{4}{5}$. Ans.

44.

7)34710 (nonary)

7)4471 seven 5 ones

7)574 (seven)²s 6 sevens

7)74 (seven)³s 3 (seven)²s

7)9 (seven)⁴s 4 (seven)³s

1 (seven)⁵s 2 (seven)⁴s

Ans. 124365 (septenary).

45. A's liability was $\frac{4}{15}$ of £10, or £2, 13s. 4d.
B's $\frac{5}{15}$ of £10, or £3, 6s. 8d.
C's $\frac{6}{15}$ of £10, or £4;
∴ A had to give C 2s. 6d., and C had to give B 5s. 6d., or A and C could give B 2s. 6d. and 3s. respectively. Ans.

46. $\frac{6}{132} - \frac{3}{93}$,

or $\frac{1}{22} - \frac{1}{31} = \frac{9}{22 \times 31}$,

and as often as this is contained in £$10\frac{4}{5}$ so many pounds has he.

$\frac{54}{5} \times \frac{22 \times 31}{9}$, or £818, 8s.

Ans.

47.

1 fifth) 5 halves
2) 5

2 tenths 25 tenths (12 times
24 „
1 tenth

12 times and 1 tenth over. Ans.

48. The trains meet one another at the rate of

$$\left(\frac{66 \times 60 \times 60}{3 \times 1760} + \frac{1320 \times 60}{1760}\right)$$

miles an hour, but there are only

$$405 - \frac{66 \times 60 \times 60}{3 \times 1760},$$

or 360 miles left in which to meet; ∴ they meet in

$$\frac{360}{45 + 45}, \text{ or } 4 \text{ hours},$$

i.e., at 2 P.M. Ans.

VII.

49. First let us take away the £2 extra that B has, then 3, 3, 5 represent the three shares.
A has $\frac{3}{11}$ of 11, or £3, B has £5, and C has £5. Ans.

50. See Hints, p. 358.

51. If 20 days of the first are worth 30 days of the other, their powers are as 3 : 2. Ans.

52. $\frac{12}{8}$ child = 1 woman.
$\frac{5}{3}$ woman = 1 man.
1 man + 2 women + 3 children = $2\frac{1}{2} + 3 + 3$ children, and these in 1 day earn 17s.; ∴ each child earns each day, 2s.; each woman, 3s.; each man, 5s. Ans.

53. In the second year he spends $\frac{4}{5}$ of $\frac{4}{5}$ of his income; ∴ his savings were $\frac{9}{25}$, and $\frac{9}{25} - \frac{1}{5}$, or $\frac{4}{25}$ of his income = £100; ∴ his income = $\frac{100 \times 25}{4}$ = £625. Ans.

54. See Hints, p. 358.

X.

73. $\frac{9}{240}$ of his income $= £52\frac{1}{2}$; $\therefore$ his income is $\frac{105 \times 240}{2 \times 9}$, or £1400,

and $\frac{£18\frac{3}{4}}{£1400} = \frac{75}{4 \times 1400}$

$= \frac{15}{4 \times 7 \times 20 \times 2}$

$= \frac{\frac{15}{14} \times 3}{20 \times 12}$;

$\therefore$ the tax was $\frac{45}{14}$,

or $3\frac{3}{14}$d. in the £. Ans.

74. There are $25 \times 4 \times 6$ hours' work to do.

$\frac{25 \times 4 \times 6}{24 \times 5} = 5$ the hrs. required.

75. This problem, as it stands, is capable of several solutions, as the following statement will show. If h and $(h-1)$ represent the hours the different lots of men worked, and the men we want to find be represented by x, we have

$24 \times 5 \times (h-1) = x \times 4 \times h$,

$$\frac{x \times 4}{24 \times 5} = \frac{h-1}{h},$$

$$\text{or } \frac{x}{30} = \frac{h-1}{h}.$$

Now, if h be any measure of 30 excepting 1, but not excepting 30 itself, we shall have an integral solution with respect to x.

E.g. Let $h = 6$.

$$\frac{x}{30} = \frac{5}{6},$$

$$\text{or } x = 25.$$

Or, again, let $h = 15$.

$$\frac{x}{30} = \frac{14}{15},$$

$$x = 28, \text{ etc.}$$

If the other number of men had been given as 24, and the fact that the respective hours they worked differed by 1, we could have found the hours thus—

$$\frac{25 \times 4}{24 \times 5} = \frac{h-1}{h},$$

$$\frac{6}{5} = \frac{h-1}{h},$$

$$\frac{6-1}{6} = \frac{h-1}{h};$$

$$\therefore h = 6.$$

76. In the 3 per cents. he has 7 cents.

In the Russian stock he has $\frac{585}{97\frac{1}{2}}$, or 6 cents.

$\therefore$ he has 13 cents., or £1300 worth of stock. Ans.

77. He gains on the English stock $£\frac{1}{2} \times 7$, or $£3\frac{1}{2}$, but loses on the Russian stock $£1\frac{1}{4} \times 6$, or $£7\frac{1}{2}$;

$\therefore$ he loses £4. Ans.

78. Let us adopt as our unit the value of the use of £1 for 1 month.

A has 1200 units,
B „ 5400 „

But these are $\frac{3}{4}$ of the units; ∴ there are 8800 units, and C has 2200 units; but he has his money in 2 months; therefore his capital was $\frac{2200}{2}$, or £1100. Ans.

79. The times vary inversely as the miles per hour he respectively travels according as he walks or runs; ∴ he would run the distance in $\frac{4}{7}$ of $3\frac{25}{60}$ hrs., or $\frac{41}{21}$ hrs., or 1 hr. $57\frac{1}{7}$'. Ans.

80.

9)5· (ninth)²s

9)4·361361 etc. ninths

·33542 etc. (septenary)

Ans. 3·33542 etc.

Note.—In dividing by 9, we had to multiply our remainders by 7 and not 10, as we were working in the septenary scale.

XI.

81.

4 gals. of gin cost him 60s.
4 gals. of water „ 0s.
1 gal. of base spirits „ 10s.
∴ 9 gals. cost him 70s.,

for which he receives $9 \times 6 \times 2\frac{5}{6}$, or 153s.;

∴ he gains 83s., or 70s.,
and 70s. : 100s. = 83 : x;

$$\therefore\ x = \frac{100 \times 83}{70} = 118\tfrac{4}{7}.$$

Ans. $118\frac{4}{7}$ p. c. Ans.

82. See Hints, p. 359.

83. If $\frac{1}{240}$ of his income = £$2\frac{1}{4}$, his income is £$2\frac{1}{4}$ × 240, or £540. Ans.

84. On every £3 I receive an income

$\frac{5}{100} + \frac{4}{100} + \frac{3}{100}$, or $\frac{12}{100}$;

∴ as often as 3600 contains $\frac{12}{100}$, so many pounds have I in each investment.

£$\frac{3600 \times 100}{12}$, or £30,000 in each. Ans.

85. He sells each sheep at $\frac{749}{700}$ of what he gave for it;

∴ his profits are $\frac{49 \times 749}{700}$.

$$749 : 100 = \frac{49 \times 749}{700} : x.$$

x = 7 p. c. Ans.

86. The tradesman receives $\frac{88}{100}$ of $\frac{125}{100}$ of buying price = $\frac{110}{100}$, or 10 p. c. Ans.

87. If 2s. $7\frac{1}{2}$d. is 20 per cent. profit on average buying price, the average buying price is

$\frac{100}{120}$ of $2\frac{7\frac{1}{2}}{12}$s., or $\frac{35}{16}$s.

$$3\text{s.} - \tfrac{35}{16}\text{s.} = \tfrac{13}{16}\text{s.},$$
$$\tfrac{35}{16} - 2\text{s.} = \tfrac{3}{16}\text{s.};$$

∴ the 3s. tea must be mixed with the 2s. tea in proportion

3 : 13. Ans.

88. $13 \times 10 = 130$,
$130 + 6 = 136$,

$$\frac{136}{7+10} = 8;$$

∴ two numbers are 8 and 5. Ans.

XII.

89. $15 + 1 = 16$,
$\sqrt{16} = 4$,
$4 \times 8 = 32$,
$32 + 4 = 36$,
$\frac{36}{2} = 18$. Ans.

It is better to do this as above; but it could be done in one effort, thus—

$$\frac{\sqrt{15+1} \times 8 + 4}{2} = 18.$$

90. Let us first take away A's extra £10, and D's extra £10, and B's extra £25; then we have £160 left to divide into 4 equal parts, of which C has 1, viz. £40. Ans.

91–96. See Hints, p. 360.

PART III.

CHAPTER XXI.

1.

$200+40+7$
$200+40+7$

$40000+40\times200+7\times200$
$\quad+40\times200+40\times40+7\times40$
$\quad\quad+7\times200+7\times40+49$

$40000+2\times40\times200+40\times40+2\times7\times200+2\times7\times40+49$

40000
16000
1600
2800
560
49

61009

2. $\sqrt{61009}$ = 247

4

44 | 210
176

487 | 3409
3409

Ans. 247.

3. $4\frac{5}{7}\times4\frac{6}{11}\times3\frac{3}{5}\times11\frac{2}{3}$
$=\frac{33}{7}\times\frac{50}{11}\times\frac{18}{5}\times\frac{35}{3}$
$=3\times10\times6\times5$
$=3\times3\times10\times10$;
∴ the square root is 30. Ans.

4. $5\frac{1}{11}\times4\frac{4}{7}\times11\frac{2}{3}\times\frac{4}{5}\times4\frac{5}{7}$
$=\frac{56}{11}\times\frac{32}{7}\times\frac{35}{3}\times\frac{4}{5}\times\frac{33}{7}$
$=8\times32\times4=32\times32$;
∴ the square root = 32. Ans.

5. $\overbrace{804609}$ (897
64

169 | 1646
1521

1787 | 12509
12509

Ans. 897.

6. $\overbrace{\cdot 02819041}$ ($\cdot 1679$
1

26 | 181
156

327 | 2590
2289

3349 | 30141
30141

Ans. $\cdot 1679$.

7. $\overbrace{12 \cdot 100000}$ ($3 \cdot 478$
9

64 | 310
256

687 | 5400
4809

6948 | 59100
55584

Ans. $3 \cdot 478 \ldots$

8. $\overbrace{4 \cdot 700000}$ ($2 \cdot 167$
4

41 | 70
41

426 | 2900
2556

4327 | 34400
30289

Ans. $2 \cdot 167 \ldots$

9. $\cdot 09 \times 144 = \frac{9 \times 144}{100}$.

The square root $= \frac{3 \times 12}{10}$, or $\frac{18}{5}$, or $3 \cdot 6$. Ans.

10. $\cdot 009 \times \cdot 004 = \cdot 000036$.
The square root is $\cdot 006$. Ans

11. $\frac{4}{5} = \cdot 800000 \ldots$

$\overbrace{\cdot 800000}$ ($\cdot 894$
64

169 | 1600
1521

1784 | 7900
7136

Ans. $\cdot 894 \ldots$

12. $\frac{1}{7} = \cdot\dot{1}42857\dot{}$.

$\cdot\dot{1}42857\dot{}$ ($\cdot 37796\ldots$

9

67 | 528
469

747 | 5957
5229

7549 | 72814
67941

487328

Ans. $\cdot 37796\ldots$

13. 640752209 (80047

64

16004 | 75222
64016

160087 | 1120609
1120609

Ans. 80047.

14. 4)169

4)42 fours 1 one

4)10 (four)^{2}s 2 fours

2 (four)^{3}s 2 (four)^{2}s

2221 (31

21

121 | 121
121

Ans. 31 (quaternary), or 13 (denary).

15. See Hints, p. 361.

16. 15^2, 16^2, 17^2, or 225, 256, 289.

17. See Hints, p. 361.

The difference of 2 squares = product of their sum and difference.

Their sum $= 2 \times 19041 + 1$, and their difference = 1, whence we get, as before, 38083. Ans.

18, 19. See Hints, p. 362.

20. Let us take any number, as 6. Its double is 12.

$$\frac{12^2}{6^2} = \frac{6^2 \times 2^2}{6^2} = 4.$$

Ans. 4 times.

21. £1 : £12278016 = 2^2 carats : x^2 carats.

$x^2 = 2^2 \times 12278016$.

We might first take out all

the factors which are square numbers, thus—

4)12278016
4)3069504
4)767376
4)191844
9)47961
5329

Now to find the square root of 5329.

$$\begin{array}{r|l} \overline{5329} & 73 \\ 49 & \\ \hline 143 \mid 429 & \\ \quad\;\; 429 & \end{array}$$

$\therefore x^2 = 2^2 \times 2^2 \times 2^2 \times 2^2 \times 3^2 \times 73^2$,

or $x = 3504$ carats. Ans.

22. $187 = 11 \times 17$.

The two numbers, added *together*, are 17, and their *difference is* 11; hence the larger number is $\frac{17+11}{2}$, or 14, and the smaller number is $\frac{17-11}{2}$, or 3.

Ans. 14 and 3.

Had we resolved 187 into the factors 187×1, we should have got 94 and 93 as the numbers which satisfy the conditions.

23. There can be as many answers to this question as there are pairs of factors of 4325.

To take first the factors 25 and 173.

The numbers are $\frac{25+173}{2}$, or 99, and $\frac{173-25}{2}$, or 74, which are the numbers given in the Hints.

Let us resolve them into the factors 5 and 865.

The numbers are $\frac{865+5}{2}$, or 435, and $\frac{865-5}{2}$, or 430.

24.

12)128881 (denary)

12)10740 twelves 1 one

12)895 (twelve)^{2}s 0 twelve

12)74 (twelve)^{3}s 7 (twelve)^{2}s

6 (twelve)^{4}s 2 (twelve)^{3}s

$\widehat{62}\widehat{70}1$ (duodenary)$\big($ $25e$

4

45 | 227
1 t1

$4te$ | 4601
4601

Ans. $25e$ (duodenary).

25. $487 = 487 \times 1$.
One number $= \frac{488}{2}$, or 244; other number $= \frac{486}{2}$, or 243.
Ans. 244 and 243. Also see Hints, p. 362.

26, 27. See Hints, p. 362.

28. The square root of
$\frac{1}{4}\{134^2 + 17^2 - 2 \times 134 \times 17\}$
$= \sqrt{\frac{1}{4}(134 - 17)^2}$
$= \frac{1}{2} \times 117$
$= 58\frac{1}{2}$. Ans.

29. $\sqrt{4^2 \times 3^2 + (\frac{7}{2})^2}$ ft.
$= \frac{1}{2}\sqrt{4^2 \times 3^2 \times 4 + 49}$ ft.
$= \frac{1}{2}$ of 25 ft.
$= 12\frac{1}{2}$ ft. Ans.

30. Let the student draw a figure.
B walks
$\sqrt{500^2 - 400^2}$,
or $\sqrt{900 \times 100}$,
or 300 yds. Ans.

31. Since A walks 400 yds. whilst B walks 300 yds., A's rate : B's rate $= 4 : 3$. Ans.

32. Let D, d, be the diameters of the two circles.
$D^2 : d^2 = 36590401 : 12809241$.
We must find the square roots of these numbers.

$$
\begin{array}{r|l}
 & \overbrace{3659040\,1}\,(6049 \\
 & 36 \\
\hline
1204 & 5904 \\
 & 4816 \\
\hline
12089 & 108801 \\
 & 108801
\end{array}
$$

$$
\begin{array}{r|l}
 & \overbrace{12809241}\,(3579 \\
 & 9 \\
\hline
65 & 380 \\
 & 325 \\
\hline
707 & 5592 \\
 & 4949 \\
\hline
7149 & 64341 \\
 & 64341
\end{array}
$$

$\therefore$ D : d = 6049 : 3579. Ans.

33. Here, again, as in **22**, there is more than one pair of circles that would differ in area by 2365 units.

$$2365 = 5 \times 473.$$

$$\text{One number} = \frac{473+5}{2} = 239;$$

$$\text{other number} = \frac{473-5}{2} = 234;$$

$\therefore$ their diameters are as

$$239 : 234.$$

See also Hints, p. 362.

34. Here, again, there are more than one pair of numbers. To find these, we proceed exactly as in **33**.

The numbers given being the same, to take other factors,

$$2365 = 2365 \times 1;$$

$\therefore$ the numbers are $\frac{2365+1}{2}$,

and $\frac{2365-1}{2}$,

or 1183 and 1182. Ans.

35. See Hints, p. 362.

36. $(1604 + 1589) \times (1604 - 1589) = 3193 \times 15$,

or 47895. Ans.

Also see Hints, p. 362.

37. The other side is

$$\sqrt{65^2 - 25^2}$$
$$= \sqrt{90 \times 40}$$
$$= \sqrt{9 \times 4 \times 100}$$
$$= 60 \text{ Ans.}$$

38. The diagonal will be

$$\sqrt{42^2 + 42^2}$$
$$= \sqrt{3528}$$

$$
\begin{array}{r|l}
 & \overbrace{3528{\cdot}000000}\,(59{\cdot}396\ldots \\
 & 25 \\
\hline
109 & 1028 \\
 & 981 \\
\hline
1183 & 4700 \\
 & 3549 \\
\hline
 & 115100
\end{array}
$$

11869	115100
	106821
118786	827900
	712716

Ans. 59·396 . . . units.

39. See Hints, p. 363.

40. See Hints, p. 363.

The diagonal of the base is $\sqrt{44^2+44^2}$, or $\sqrt{3872}$, or $\sqrt{2}\times 44$.

And the diagonal of the cube is $\sqrt{3872+1936}$, or $\sqrt{3}\times 44$.

5808·00 . . . (76·2102 . . .

49

146	908
	876
1522	3200
	3044
15241	15600
	15241
152402	359000

Ans. 76·2102 . . .

41. See Hints, p. 363.

42. See Hints, p. 363.

We have to find $\sqrt{16^2-8^2}$

$= \sqrt{24\times 8} = \sqrt{192}$,

or $\sqrt{3}\times 8$.

192·00 . . . (13·856 . . .

1

23	92
	69
268	2300
	2144
2765	15600
	13825
27706	177500
	166236

11264

If we have tables, we can work thus—

$\sqrt{3} = 1{\cdot}7320508$

8

13·8564064

Ans. 13·85640 . . . feet.

43, 44. See Hints, p. 363.

45. The diagonal is

$\sqrt{216^2+195^2}$

$= \sqrt{9(72^2+65^2)}$

$= 3\sqrt{9409}$

$= 3\times 97$;

∴ He saves $216+195-291$, or 120 yards. Ans.

46. See Hints, p. 364.

(1) To find the height of wall reached by end of ladder

$\sqrt{65^2-33^2} = \sqrt{98\times 32}$

$= \sqrt{2\times 49\times 2\times 16} = 56$;

∴ the wall is $56+7$, or 63 feet high.

To solve the other part we have to find

$$\sqrt{65^2 - 63^2} = \sqrt{128 \times 2}$$
$$= \sqrt{2 \times 64 \times 2} = \underline{16 \text{ feet.}} \text{ Ans.}$$

47. $DB = \sqrt{5^2 - 3^2}$
$= \sqrt{8 \times 2}$
$= \underline{4}$. Ans.

48. Since the sides of similar triangles are in the duplicate ratio of their homologous (corresponding) sides, we have—

$S^2 : s^2 = 132496 : 5322249$

$\overbrace{132496}(364$
9

$66 \mid 424$
396

$724 \mid 2896$
2896

$\overbrace{5322249}(2307$
4

$43 \mid 132$
129

$4607 \mid 32249$
32249

$\therefore S : s = \underline{364 : 2307}$. Ans.

49. $\overbrace{99999}(317$
9

$61 \mid 99$
61

$627 \mid 3899$
4389

We must therefore add to

99999
490

100489

Ans. $\underline{100489}$.

50. $7)201$ (quinary)

$7)12$ (sevens) 2 ones

1 (seven)^{2}s 0 seven

$\therefore$ 201 (quinary) = 102 (septenary).

Hypotenuse required is

$\sqrt{102^2 + 125^2}$
$= \sqrt{10404 + 16324}$
$= \sqrt{30031}$

$\overbrace{30031}(151$
1

$25 \mid 200$
164

$331 \mid 331$
331

Ans. 151 (septenary), or ☰ (denary).

CHAPTER XXII.

1 to **4**. See Hints, p. 364.

5. $34\frac{3}{4} \times 26\frac{1}{2} \div 9 \times 4\frac{1}{2}$ will give us the cost in shillings, or $\frac{139}{4} \times \frac{53}{2} \times \frac{1}{9} \times \frac{9}{2}$s., or $\frac{73667}{16}$s., or £23, os. $5\frac{1}{4}$d. Ans.

6. $14 \times (74\frac{1}{2} + 36\frac{2}{3}) \div 2\frac{1}{3}$ will give us the linear feet in the paper, and this $\times \frac{1}{2 \times 20}$ will give us the cost in pounds, or

$$£14 \times 111\tfrac{1}{6} \times \tfrac{3}{7} \times \frac{1}{2 \times 20}$$

$$£\frac{14 \times 667 \times 3}{6 \times 7 \times 2 \times 20},$$

or £16, 13s. 6d. Ans.

7. $14\frac{5}{12} \times 18\frac{7}{12}$ sq. ft., or 29 sq. yds. 6 ft. 131 in.

8. From **7** we can get the area as $\frac{38579}{144}$ sq. ft., or $\frac{38579}{144 \times 9}$ sq. yds., and this $\div \frac{3}{4}$ is $\frac{38579}{144 \times 9} \times \frac{4}{3}$ yds., and this at 4s. 6d. a yd. is $£\frac{38579 \times 4 \times 9}{144 \times 9 \times 3 \times 2 \times 20}$, or £8, 18s. $7\frac{5}{18}$d. Ans.

9. The whole area is 5 ft. broader and longer than the central portion, and its area is $19\frac{5}{12} \times 23\frac{7}{12}$ sq. ft., or $\frac{65939}{144}$ sq. ft., and this less $\frac{38579}{144}$ will give us the area of the border, viz. $\frac{27360}{144}$ sq. ft.,

or 21 sq. yds. 1 ft. Ans.

10. The area of the walls is $\frac{35}{2}(123 + 93)$ sq. ft.,

or $\frac{35 \times 216}{2 \times 9}$ sq. yds.,

and the length of the paper is

$$\frac{35 \times 216 \times 4}{2 \times 9 \times 3} \text{ yds.},$$

and this will cost

$$\frac{35 \times 216 \times 4 \times 9}{2 \times 9 \times 3 \times 4 \times 12}\text{s.},$$

or £5, 5s. Ans.

11. To do this in one effort,

$$18\tfrac{3}{4} \times 17\tfrac{1}{2} \times \tfrac{1}{2} \times 4\tfrac{3}{4}\text{s.},$$

or $£\frac{75 \times 35 \times 19}{4 \times 2 \times 2 \times 4 \times 20}$,

or £38, 19s. $3\frac{9}{11}$d. Ans.

12. $£12 \times (57 + 37\frac{1}{2}) \div 1\frac{3}{4} \div 3 \times \frac{1}{8}$,

or $£\frac{12 \times 189 \times 4}{2 \times 7 \times 3 \times 8}$,

or £27. Ans.

13. $10 \times (50 + 37) \times \frac{1}{2} \times \frac{1}{3} \times \frac{3}{12}$s.,

or $£\frac{10 \times 87 \times 3}{2 \times 3 \times 12 \times 20}$,

or £1, 16s. 3d. Ans.

14. $£\frac{50 \times 3 \times 50 \times 5}{1 \times \frac{3}{4} \times 12}$,

or $£\frac{50 \times 3 \times 50 \times 5 \times 4}{3 \times 12}$,

or £4166, 13s. 4d. Ans.

15. $15 \times 4 \times 15$ sq. ft.

Subtracting the door and window,

$\frac{(900 - 24 - 21)}{9}$ sq. yds.;

$\therefore$ cost is $£\frac{95 \times 1\frac{1}{6}}{20}$,

or £5, 10s. 10d. Ans.

16. 9)87001401 6 sq. ft.

9666822 4 sq. yds.

9666822 4 (9832
81

188 | 1566
1504

1963 | 6282
5889

19662 | 39324
39324

And 9832 yds.
= 5 mls. 1032 yds. Ans.

17. The student is recommended to draw a figure, and let the top and bottom be the full 3 ft. 6 in. × 2 ft. 6 in.

The front and back the full width, and $\therefore$ 3 ft. 6 in. × 1 ft. 6 in., and the sides only 2 ft. 3 in. × 1 ft. 6 in. Then the planking used is double these products, viz.,

$2(3\frac{1}{2} \times 2\frac{1}{2} + 3\frac{1}{2} \times 1\frac{1}{2} + 2\frac{1}{4} \times 1\frac{1}{2})$ sq. ft.,

or $2(\frac{35}{4} + \frac{21}{4} + \frac{27}{8})$ sq. ft.,

or $\frac{70 + 42 + 27}{4 \times 9}$ sq. yds.,

or 3 sq. yds. 7 ft. 108 in. Ans.

18. $21\frac{2}{3} \times 16\frac{1}{2} \div 2\frac{1}{4} \div \frac{1}{8} \times 3\frac{3}{8}$s.,

$= \frac{65 \times 33 \times 4 \times 27}{3 \times 2 \times 9 \times 3 \times 8}$s.,

or £8, 18s. 9d. Ans.

19. The length of the paper is $10 \times (2 \times 15 + 2 \times 13) \div 1\frac{1}{6}$ ft.

or $\frac{10 \times 56 \times 6}{7}$ ft., or 480 ft.,

and the cost will be

$£\frac{480 \times 1\frac{1}{2}}{12 \times 20}$, or £3. Ans.

20. 3 ac. 1 ro. 38 po. $20\frac{1}{2}$ yds.

4

13 ro.
40

558 po.
$30\frac{1}{4}$

$16760\frac{1}{2}$

$16760\frac{1}{2}$
$139\frac{1}{2}$

16900 sq. yds.

∴ the side of the field is 130 yds.; ∴ the large field is $(130 + 390) \times (130 + 65)$ sq. yds., or 520×195 sq. yds.,
or 20 ac. 3 ro. 32 po. 2 yds. Ans.

21, 22. See Hints, p. 365.

23. $122\frac{1}{2}$ ac. $= 122\frac{1}{2} \times 4840$ sq. yds. $= 245 \times 2420 = 5 \times 49 \times 5 \times 11 \times 11 \times 2 \times 2$; ∴ side is $5 \times 7 \times 11 \times 2$ yds.
or 770 yds. Ans.

24, 25. See Hints, p. 365.

26. The side required is

$$\sqrt{\left(77\frac{1\frac{3}{4}}{3}\right)^2 + \left(7\frac{2\frac{1}{3}}{3}\right)^2} \text{ yds.},$$

or $\sqrt{\left(\frac{931}{12}\right)^2 + \left(\frac{70}{9}\right)^2}$ yds.,

or $\frac{1}{36}\sqrt{7879249}$ yds.

$$\begin{array}{r|l} & \widehat{78}\,\widehat{79}\,\widehat{24}\,\widehat{9}\,(2807 \\ & 4 \\ \hline 48 & 387 \\ & 384 \\ \hline 5607 & 39249 \\ & 39249 \\ \hline \end{array}$$

and $\frac{5607}{36}$ yds. are 77 yds. 2 ft. 11 in. Ans.

27.

$$\begin{array}{r|l} & \widehat{82}\,\widehat{44}\,\widehat{64}\,(908 \\ & 81 \\ \hline 1808 & 14464 \\ & 14464 \\ \hline \end{array}$$

Ans. 908 yds.

28. The whole court contains 120×90 sq. ft. The central portion contains 100×70 sq. ft.; ∴ the path contains $120 \times 90 - 100 \times 70$ ft., or 3800 sq. ft., or $\frac{3800}{9}$ sq. yds.; and these yds. at 4s. 6d. each will cost $£\frac{3800 \times 9}{9 \times 2 \times 20}$, or £95.

The turfing will cost $£\frac{100 \times 70 \times 13}{100 \times 2 \times 20}$, or £22, 15s.;

∴ the entire cost will be £117, 15s. Ans.

29. See Hints, p. 366.

Let us solve it by proportion,

$$\left.\begin{array}{r} 69\frac{1}{3} \text{ ft.} : 22\frac{1}{6} \text{ ft.} \times 3 \\ 6\frac{3}{4} \text{ ft.} : 3\frac{1}{3} \text{ ft.} \times 3 \\ 2 \text{ value} : 3 \text{ value} \end{array}\right\} = £49\ \frac{14\frac{1}{2}}{20} : £x$$

$$x = \frac{111 \times 3 \times 10 \times 3 \times 3 \times 1989 \times 3 \times 4}{5 \times 3 \times 40 \times 208 \times 27 \times 2}, \text{ or } \underline{£106, 2s.\ 10\tfrac{1}{2}d.} \text{ Ans.}$$

30. This is given at once. Get the following fraction

$$\frac{54 \times 3 \times 21 \times 3}{13\frac{1}{2} \times \frac{10\frac{1}{2}}{12}} \text{ planks,}$$

$$\text{or } \frac{54 \times 3 \times 21 \times 3 \times 2 \times 8}{27 \times 7},$$

or 864.

Each plank contains

$$13\tfrac{1}{2} \times \frac{10\frac{1}{2}}{12} \text{ sq. ft.,}$$

$$\text{or } \frac{27 \times 7}{2 \times 8} \text{ sq ft.;}$$

and 864 times this at $5\frac{1}{2}$ per sq. foot will cost

$$£\frac{864 \times 27 \times 7 \times 11}{2 \times 8 \times 2 \times 12 \times 20}$$

or $\underline{£233, 17s.\ 9d.}$ Ans.

31.

ac.	ro.	po.
9	3	8·16
4		
39	ro.	
40		
1568·16	po.	

1568·16 po. (39·6

9

69 | 668
| 621

786 | 4716
| 4716

Side of square 39·6 poles, or 217·8 yds.; ∴ side of central square is 210 yds.; ∴ area of central square = 44100 sq. yds.; ∴ area of running path = 1568·16 po. − 44100 sq. yds., or 3336·84 sq. yds., and this will cost $£\frac{333684 \times 4}{100 \times 12 \times 20}$,

or £55, 12s. $3\frac{9}{25}$d.

and the turf plat cost

$£\frac{44100 \times 11}{100 \times 2 \times 20}$, or £121, 5s.;

∴ entire cost is

$\underline{£176, 17s.\ 3\frac{9}{25}d.}$ Ans.

32. See Hints, p. 366.

33.

ac.	ro.	po.	yds.
3	1	13	$5\frac{3}{4}$
4			
13	ro.		
40			
533	po.		
$30\frac{1}{4}$			
15995	$\frac{3}{4}$		
1331	$\frac{1}{4}$		
16129	yds.	(127	
1			
22 | 61			
44			
247 | 1729			
1729			

Ans. 127 yds.

34. If the area $= 5\frac{5}{8}$ ac. the side is $\sqrt{\frac{45\times4\times40}{8}}$ po., or 30 po.; ∴ he walks 120 po.

po. po. min. min.

$320 : 120 = 10\frac{2}{3} : x.$

$x = \frac{120\times32}{320\times3}$ min.

$= 4$ min. Ans.

35. See Hints, p. 366.

36. The area of the walls is $11\frac{2}{3}(42+31\frac{1}{2})$, or $\frac{35\times147}{3\times2}$ sq. ft., and the area of the ceiling is $21\times15\frac{3}{4}$ sq. ft., or $\frac{21\times63}{4}$ sq. ft.; then we have this proportion,

sq. ft. sq. ft. £ £

$\frac{35\times147}{3\times2} : \frac{21\times63}{4} = \frac{77}{8} : x$

$x = \frac{21\times63\times77\times3\times2}{4\times8\times35\times147}$

$= £3,\ 14s.\ 3d.$ Ans.

37 to **40.** See Hints, p. 367.

41. Since $1+2\frac{1}{3}=3\frac{1}{3}$, we must divide 100 by $3\frac{1}{3}$, which gives us 30; ∴ the parts are 30 and 70. Ans.

42. See Hints, p. 367.

43. This can only be done arithmetically by trial.

$1\times1+9=10$, which is not a square number.

$2\times2+9=22$, which is not a square number.

But $3\times3+9=36$, which is a square number. Ans. 36 sq. ft.

44.

chs.	links.
3	50
6	
21	·00 sq. chs.

and 21 sq. chs. $= 2{\cdot}1$ acres, or 2 ac. 0 ro. 16 po. Ans.

45, 46. See Hints, p. 367.

47. This is found by dividing 200 ac. by 220 yds. Thus

$$\frac{\text{200 ac.}}{\text{220 yds.}}$$

$$\frac{200 \times 4 \times 40 \times 121}{220 \times 4},$$

or 4400 yds.,

or $2\frac{1}{2}$ miles. Ans.

48. Since the figures vary according to the squares of their homologous sides, we have this proportion—

$$3^2 : 12^2 = 1 : x.$$

$$x = \frac{144 \times 1}{9} = 16.$$

49. $\frac{3}{4} \times 1760 \times 3 \times 4$ sq. ft.,

or 9×1760 sq. ft.,

or 1760 sq. yds.,

or $\frac{1760}{4840} = \frac{4}{11}$ of an acre.

50. See Hints, p. 367.

CHAPTER XXIII.

1. See Hints, p. 368.

2. A figure would make this plain, that the 8 beams would be 1 yd. long, 1 ft. wide, and 1 ft. thick.

3, 4. See Hints, p. 368.

5. $\frac{270 \times 3 \times 16 \times 5 \times 4 \times 3 \times 4}{3 \times 1 \times 1 \times 4}$,

or 259200 bricks. Ans.

6. The student is recommended to draw a figure.

The top and bottom each contain $6\frac{1}{6} \times 3\frac{2}{3} \times \frac{1}{12}$ cub. ft. The front and back each contain $6\frac{1}{6} \times 1\frac{5}{6} \times \frac{1}{12}$ cub. ft.; and the two ends each contain $3\frac{1}{2} \times 1\frac{5}{6} \times \frac{1}{12}$ cub. ft.;

$\therefore$ the wood used was

$2(\frac{37}{6} \times \frac{11}{3} \times \frac{1}{12} + \frac{37}{6} \times \frac{11}{6} \times \frac{1}{12} + \frac{7}{2} \times \frac{11}{6} \times \frac{1}{12})$ cub. ft.,

or $2\left(\frac{407}{6.3.12} + \frac{407}{6.6.12} + \frac{77}{2.6.12}\right)$ cub. ft.,

or $2\left(\frac{814 + 407 + 231}{6.3.12.2}\right)$ cub. ft.

or $\frac{1452}{6.3.12}$ cub. ft.,

or $6\frac{13}{18}$ cub. ft. Ans.

7. $30\frac{3}{4} \times 16\frac{7}{12} \times 6\frac{1}{3}$ cub. ft.

$= \frac{123}{4} \times \frac{199}{12} \times \frac{19}{3}$

$= \frac{155021}{48}$,

$= 3229$ cub. ft. 1044 in. *Ans.*

8, 9. See Hints, p. 368.

10. $\frac{36\times 9\times 12}{\frac{3}{4}\times\frac{9}{24}\times\frac{1}{4}}$ is the number of bricks, and this is

$$=\frac{36\times 9\times 12\times 4\times 24\times 4}{3\times 9},$$

or 55296 bricks. Ans.

11. $\frac{25\times 3\times 15\times 1\frac{7}{8}}{\frac{3}{4}\times\frac{9}{24}\times\frac{1}{4}}$ bricks,

or $\frac{25\times 3\times 15\times 15\times 4\times 24\times 4}{8\times 3\times 9}$,

or 30000 bricks. Ans.

12. The answer is given by the expression

$\sqrt[3]{154\frac{11}{12}\times 70\frac{7}{12}\times 53\frac{1}{12}}$,

or $\frac{1}{12}\sqrt[3]{1859\times 847\times 637}$,

or $\frac{1}{12}\sqrt[3]{11^3\times 7^3\times 13^3}$,

or $\frac{1}{12}$ of 1001 ft.,

or 83 ft. 5 in. Ans.

13. $\frac{25\times 20}{8}=\frac{25\times 5}{2}$,

$\frac{125}{2}=62\frac{1}{2}$,

or 62 children.

$\frac{25\times 20\times 10\frac{1}{3}}{62\frac{1}{2}}$ will give the cub. extent assigned to each child $\frac{25\times 20\times 31\times 2}{125\times 3}$, or $82\frac{2}{3}$; but only counting 62 children, the cub. extent of each child is $\frac{25\times 20\times 31}{62\times 3}$, or $83\frac{1}{3}$.

14. $10\times 30\times 1\times\frac{1}{12}\times 20$ lbs. $=500$ lbs.,

or 4 cwts. 1 qr. 24 lbs. Ans.

15. The weight is

$9\frac{1}{2}\times 2\frac{1}{4}\times 2\times 2\cdot 716\times 1000$ oz.,

or $\frac{19\times 9\times 2\times 2716}{2\times 4}$ oz.,

or $19\times 9\times 679$ oz.,

or 3 tons 4 cwts. 3 qrs. 4 lbs. 13 oz. Ans.

16. The value is $£5\frac{1}{4}\times 2\frac{1}{3}\times 1\frac{1}{8}\times\frac{84}{20}$,

or $£\frac{21\times 7\times 7\times 84}{4\times 3\times 6\times 20}$,

or £60, 0s. 6d. Ans.

17. This is a question in Simple Proportion.

$\frac{21\times 7\times 7}{4\times 3\times 6}$ c. ft. : $\frac{7\times 7\times 5}{4\times 6}$ c. ft.

$=1$ cwt. : x cwt.

$x=\frac{7\times 7\times 5\times 4\times 3\times 6}{4\times 6\times 21\times 7\times 7}$ cwt.,

or $\frac{5}{7}$ cwt., or 2 qrs. 24 lbs. Ans.

18. $\frac{1728}{48\frac{2000000}{1}}$ inches, or $\frac{1}{250000}$ of an inch. Ans.

19. Cub. dec. $=3\cdot 937\times 3\cdot 937\times 3\cdot 937$ cub. in., and it or a kilogramme weighs $3\cdot 937\times 3\cdot 937\times 3\cdot 937\times 252\cdot 45$ grs., or $\frac{3937\times 3937\times 3937\times 25245}{1000000000\times 7000}$ lbs.

Since we are only asked to obtain the result correct to three places of decimals, for 3937, we may venture to write 3936, and for 25245, 25250; and the fraction thus modified and cancelled becomes

$$\frac{984 \times 984 \times 984 \times 505}{25 \times 25 \times 25 \times 100000 \times 140}.$$

Again, we may venture, without any fear of affecting the fourth place of decimals, to neglect the 4 in one 984, calling it 980, and the 5 in 505, calling it 500, and this reduces the fraction to

$$\frac{7 \times 984 \times 984}{25 \times 25 \times 25 \times 200}$$
$$= \frac{7 \times 984 \times 984 \times 32}{25 \times 25 \times 25 \times 64 \times 100}$$
$$= 2 \cdot 16889344,$$

or 2·168 lbs. Ans.

20. A cub. metre contains

$$\frac{394 \times 394 \times 394}{1000} \text{ cub. in.};$$

∴ a litre will contain

$$\frac{394 \times 394 \times 394}{1000 \times 1000} \text{ cub. in.},$$

or 61·162984 cub. in. Ans.

21. See Hints, p. 369.

22. This is found by dividing £407, 3s. 2¾d., or £$\frac{390875}{960}$ by $29\frac{1}{2} \times 12\frac{1}{2} \times 10$,

or £$\frac{390875 \times 2 \times 2}{960 \times 59 \times 25 \times 10}$,

or 2s. 2½d. Ans.

23 to **25.** See Hints, p. 369.

26.

1	0	00	10546683057(2193
	2	4	8
	2	4	
	2	8	
	4		
	2		

1	60	1200	2546
	1	61	1261
	61	1261	
	1	62	
	62		
	1		

1	630	132300	1285683
	9	5751	1242459
	639	138051	
	9	5832	
	648		
	9		
1	6570	14388300	43224057
	6573	19719	43224057
		14408019	

Ans. 2193.

27. 99 tons contain 1000 oz. $\frac{99 \times 20 \times 112 \times 16}{1000}$ times; ∴ the cub. contents of the water must not be larger than this number of feet; ∴ the depth must not be greater than $\frac{99 \times 20 \times 112 \times 16}{1000 \times 48 \times 42}$ ft., or $1\frac{19}{35}$ ft. Ans.

28.

10·8
9·4
368
800
83·68
8·9
62800
56454
607·140 (duodenary)

Reducing this to denary, it becomes 871·1 cub. ft., or 871 cub. ft. 192 in. Ans.

29. See Hints, p. 369.

30. The interior space of the bin is $20 \times 8 \times 2218 \cdot 192$ cub. in., or 354910·720 cub. in. We must find the cube root of this.

1	0	00	354910·720 (70·8 . .
	7	49	343
	7	49	
	7	98	
	14		
	7		
1	2100	1470000	11910720
	8	16864	11894912
	2108	1486864	

Ans. 70·8, etc.

31.

94·50 chs.
1·05 chs.

47250
9450

99·225 sq. chs.
·315

496125
99225
297675

31·255875

or 31 cub. chs. 255875 lks. Ans.

32 to 34. See Hints, p. 369.

35. The cubic content of the box is $4\frac{1}{2} \times 2\frac{1}{3} \times 2\frac{1}{3}$, and each book contains $\frac{3}{4} \times \frac{1}{3} \times \frac{1\frac{1}{2}}{12}$ cubic feet; ∴ the box will contain $\frac{9 \times 7 \times 7 \times 4 \times 3 \times 8}{2 \times 3 \times 3 \times 3}$, or 784; ∴ 1000 − 784, or 216 must be left out. Ans.

36. This problem as it at present stands is indeterminate. We must assume some height. The area must be 100 sq. in., but the heights might be any 2 numbers whose difference is 10, as 2 and 12. Let us assume one as 3, then the other is 13, and these numbers will express the ratio between their cubic contents.

37.

11·16
6·68

7500
5690
5690

60·2288
91·8

40754
5612

511134 cub. ft.,

or 31 cub. yds. 13 ft. 1480 in. Ans.

38. The original cub. content was 56 × 9 × 20 cub. feet; after it is shaped it only contains $\frac{3}{4}$ of 56 × 9 × 20 cub. ft., and if it weighs 108 tons, 1 ft. weighs $\frac{108 \times 4}{3 \times 56 \times 9 \times 20}$ tons, or $\frac{108 \times 4 \times 20 \times 112}{3 \times 56 \times 9 \times 20}$ lbs., or 32 lbs. Ans.

39.

4·37
2·63

1079
2196
872

11049
8·9

816369
728320

711·99569

Ans. 94 cub. ft. 1409$\frac{9}{16}$ in.

40. This can be done in one effort. The cost is

$$£96 \times \frac{1}{8 \times 12} \times \frac{11000}{16} \times \frac{8}{12 \times 20},$$

or £22, 18s. 4d. Ans.

41. Let the bar be 1 sq. in. The weight of the mercury and the air is $29{\cdot}388 \times \frac{1}{2}$ lb., or 14·694 lbs., and each cub. in. of air weighs ·31 grs., and 14·694 lbs. contains ·31 grs. $\frac{14{\cdot}694 \times 7000}{{\cdot}31}$ times; ∴ the height of the column of air is 331800 in., or 5 mls. 416 yds. 2 ft.

42.

58*te*
6*e*47

34245
1*te*78
53201
2*t*556

33*t*99*e*05. Ans.

43. See Hints, p. 370.

£3411 9s. 1 guineas.
20 21

68229s. 21s.
21

68229
136458

1432809

1432809(1197
1

21 | 43
21

229 | 2228
2061

2387 | 16709
16709

1197s. or £59, 17s., or 57 guineas. Ans.

44. Since the contents of similar solids vary as the cube of their corresponding measurements, we have this proportion—

$$1 : 729 = 7^3 : x^3$$
$$x^3 = 9^3 \times 7^3,$$
or $x = 63$. Ans.

45, 46. See Hints, pp. 370, 371.

47. The amount of £250 for 3 years being £ 400, 8s. 0·78d., if we divide this by £250 we have the cube of the amount of £1 for 1 year. Reducing to pounds and dividing by £250 or £$\frac{1000}{4}$ we have

12)·78d.

20)8·065s.

£400·40325
4

1·60161300

and of this quotient, which is an abstract number, we must find the cube root.

1	0	00	1·601	613	1·17
	1	1	1		
	—	—			
	1	1			
	1	2			
	—				
	2				
	1				
1	30	300	601		
	1	31	331		
	—	—			
	31	331			
	1	32			
	—				
	32				
	1				
1	330	36300	270613		
	7	2359	270613		
	—	—			
	337	38659			

∴ the rate was 17 p. c.

48. $\frac{810 \times 14 \times 1\frac{1}{2}}{\frac{3}{4} \times \frac{4\frac{1}{2}}{12} \times \frac{3\frac{1}{2}}{12}}$ bricks,

or $\frac{810 \times 14 \times 3 \times 4 \times 8 \times 24}{2 \times 3 \times 3 \times 7}$,

or 207360 bricks. Ans.

49. The cost of the larger bricks would be

$$£\frac{207360 \times 7\frac{1}{2}}{1000 \times 20},$$

and the cost of the smaller would have been

$$\frac{9}{8} \text{ of } £\frac{207360 \times 15}{1000 \times 20 \times 2},$$

and the difference between these is

$$\frac{1}{8} \text{ of } £\frac{207360 \times 15}{1000 \times 20 \times 2},$$

or £9, 14s. $4\frac{4}{5}$d. Ans.

CHAPTER XXIV.

1. The area is

$\frac{1}{2}.2.\sqrt{3}$ sq. ft.,

or 1·7320508 . . sq. ft.,

or 1 ft. 105·41531 . . in. Ans.

2. A figure will show that *this area is one* fourth of the larger, the triangles being equilateral, whose sides are 1 ft. each.

4)1 ft. 105·41531

·62·35382 . . in. Ans.

3. $\frac{1}{2} \times 8 \times 10$, or 40 sq. ft. Ans.

4. If the student draw rectangles = the two triangles, he will at once see that the larger parallelogram is $\frac{10}{2}$ or 5 ft. longer than the smaller; ∴ the altitude of the larger triangle is 5 ft. more than that of the smaller. Ans.

5. See Hints, p. 372.

6. A figure will show that the triangle is $\frac{1}{8}$ of the square, or $\frac{1}{8}$ of 64 sq. ft., or 8 sq. ft. Ans.

7. To apply formula in paragraph 4 or 5 of this chapter—

3·15	5 − 3·15 = 1·85
4·17	5 − 4·17 = ·83
2·68	5 − 2·68 = 2·32

2)10·00 or 2s.

5·00 or s.

$$\text{area} = \sqrt{5 \times 1{\cdot}85 \times {\cdot}83 \times 2{\cdot}32}$$
$$= \tfrac{1}{10000}\sqrt{5^2 \times 2^2 \times 178118}$$
$$= \tfrac{1}{100} \times 422{\cdot}04 \text{ . . sq. chs.}$$
$$= 4{\cdot}2204 \text{ . . sq. chains}$$
$$= 42204 \text{ . . acres}$$
$$\cdot 42204 \text{ . . ac.}$$
$$4$$
$$1{\cdot}68816 \text{ ro. Ans.}$$

8. If the student draw a figure he will see that the largest *rectangle with the* given base that can be cut is one whose middle point of base is at the middle point of base of triangle, and this rectangle is half the triangle.

Its area, both of it and of what is left, is

$\frac{1}{2}$ of $\frac{1}{4} \times 6 \times 6 \times \sqrt{3}$ sq. ft.,
or $\frac{9}{2} \times 1{\cdot}7320508$ sq. ft.,
or 7·7942286 sq. ft. Ans.

9. Since a hexagon contains 6 equilateral triangles, whose sides are equal to that of the hexagon, the area required is

$6 \times \frac{1}{2} \times 2 \times \sqrt{3}$ sq. chs.,
or $6 \times 1{\cdot}7320508$ sq. chs.,
or 10·3923048 sq. chs.,
or 1 ac. 0 ro. 6·296 . . . po. Ans.

10. The student is recommended to draw a figure, and join each angular point with the next but 2; he will then divide the figure into 1 square (whose sides are = that of the octagon), 4 triangles which together are = the middle square, and 4 rectangles whose sides are as $1 : \sqrt{2}$; ∴ the area is

$$16 + 16 + 4 \times 4 \times \frac{4}{\sqrt{2}} \text{ sq. chs.}$$

$$\text{but } \frac{4}{\sqrt{2}} = 2\sqrt{2};$$

∴ the area is 77·2548352 sq. chs., or 7 ac. 2 ro. 35·277 . . po. Ans.

We might write the area down thus—

$4^2(1 + 1\frac{1}{2} + 2\sqrt{2})$,
or $4^2 \times 4{\cdot}8284272$,
or $s^2 \times 4{\cdot}8284272$,
as given in tables for finding the areas of regular figures.

11. $\widehat{4840})69{\cdot}57$

36

129 | 1240
1161

1385 | 7900
6925

13907 | 975 00
97349

or very nearly 69·57 yds. Ans

12. 2425 $3162 - 2425 = 737$
2418 $3162 - 2418 = 744$
1481 $3162 - 1481 = 1681$

2)6324 or 2s.

3162 or s.

area $= \sqrt{3162 \times 737 \times 744 \times 1681}$
$= \sqrt{6 \times 31 \times 17 \times 737 \times 6 \times 4 \times 31 \times 1681}$
$= 6 \times 31 \times 2\sqrt{17 \times 737 \times 1681}$
$= 372\sqrt{17 \times 737 \times 17 \times 98}$ nearly
$= 372 \times 17\sqrt{737 \times 98}$
$= 372 \times 17 \times 268{\cdot}7$
$= 1699258{\cdot}8$ sq. links
$= 169{\cdot}92588$ sq. chs.
$=$ 16 ac. 3 ro. 30·81 . . po. nearly. Ans.

13. 20 $35 - 20 = 15$
21 $35 - 21 = 14$
29 $35 - 29 = 6$

2)70 or 2s.

35 or s.

$$\text{area} = \sqrt{35 \times 15 \times 14 \times 6} \text{ sq. ft.}$$
$$= \sqrt{5 \times 7 \times 5 \times 3 \times 7 \times 2 \times 3 \times 2}$$
$$= 5 \times 7 \times 3 \times 2$$
$$= 210 \text{ sq. ft. Ans.}$$

If it is known that the largest angle is a right angle

$$\text{area} = \tfrac{1}{2} \times 20 \times 21$$
$$= 210 \text{ sq. feet as before.}$$

14. The side of the square
$= \frac{1}{4}$ of $\frac{12}{60}$ of 5 miles
$= \frac{1}{4}$ mile
area $= \frac{1}{16}$ sq. mile
$= \frac{1}{16}$ of 640 acres
$= 40$ acres. Ans.

15. See Hints, p. 372.

16. A figure will show that I take away one half; ∴ the part left is $\frac{1}{2}$ of 10^2 sq. in., or 50 sq. in. Ans.

17. See Hints, p. 372.

18. 10×4, or 40 sq. ft. Ans.

19. The altitude of the parallelogram is $\frac{1}{2} \times 7 \times \sqrt{3}$;
∴ the area $= \frac{1}{2} \times 7 \times \sqrt{3} \times 10$
$= 35 \times 1{\cdot}7320508$
$= 60{\cdot}62177 \ldots$ sq. ft. Ans.

20 to **24.** See Hints, pp. 372, etc.

25. $6 \times \frac{1}{2}(16 + 24)$,
or 120 sq. ft. Ans.

26. Let ABCD be the trapezoid, of which AD = 16, AB = 10, and BC = 24. From A draw AE perpendicular to BC, then

$$\text{BE} = \sqrt{10^2 - 6^2}$$
$$= 8;$$
$$\therefore \text{EC} = 16 = \text{AD};$$

∴ AECD is a rectangle, and DC is 6 ft. Ans.

27. Use the same figure as in **26**, and see Hints, p. 375.

28. The area $= 2 \times 4$ sq. ft., or 8 sq. ft.

If s = side of equilateral area
$= \frac{1}{4} \cdot s^2 \cdot \sqrt{3}$;

$$\therefore s^2 = 8 \times \frac{4}{\sqrt{3}}$$
$$= \tfrac{1}{3}(8 \times 4\sqrt{3})$$
$$= 18{\cdot}4752 \ldots$$

or $s = 4{\cdot}29$ ft. Ans.

29. We can proceed as in **10**, or multiply $(1\frac{1}{4})^2$ by 4·8284272.

16)4·8284272

·3017767

25

7·54441 sq. ft. Ans.

30. Let the sides of the triangle contain linear units, 8, 16, 18.

Its area contains

$\sqrt{21 \times 13 \times 5 \times 3}$ sq. units,

or $3\sqrt{455}$ sq. units,

or $3 \times 21{\cdot}17$ sq. units,

63·51.

Each square unit contains $\frac{3383}{6351}$ sq. in., and each linear unit $\sqrt{\frac{3383}{6351}}$ in., or $\frac{58{\cdot}1}{79{\cdot}7}$ in.;

∴ the sides are $\frac{4648}{797}$ in., $\frac{9296}{797}$ in., and $\frac{10458}{797}$ in. Ans.

31. By the aid of tables we can find this very easily.

Area of octagon

$= 36 \times 4{\cdot}8284272$ sq. ft.,

area of hexagon

$= 36 \times 2{\cdot}5980762$ sq. ft.,

and the difference

$= 36 \times 2{\cdot}230346$ sq. ft.,

or 80·29 . . . sq. ft. Ans.

32. Divide 80·29 by 2·230346, *and find* the square root of the *quotient*.

33. If s = length of the side, we have

$s^2 + \frac{1}{4} \times s^2 \times \sqrt{3} = 22{\cdot}92816$,

$s^2(1 + {\cdot}4330127) = 22{\cdot}92816$;

∴ s^2 = very nearly 16 sq. in.;

∴ s = 4 in. Ans.

34 to **37**. See Hints, p. 377.

38. $\frac{1}{2}$ of $25 \times (37 + 13)$

$= 625$ sq. chs.

$= 62{\cdot}5$ ac.,

or 62 ac. 2 ro. Ans.

39. The field is an isosceles triangle whose altitude is $\sqrt{5^2 - 3^2}$, or 4 chs. Area = $\frac{1}{2} \times 6 \times 4$ sq. chs. = $1\frac{1}{5}$ ac., and this at £4 a sq. po. will cost $\frac{6 \times 4 \times 40 \times 4}{5}$, or £768. Ans.

40. From each strip of the square, $\frac{1}{2}\sqrt{3}$ in. wide, I can cut 36 + 35 triangles, viz. 36 with their bases on the side of the square, and 35 with their vertices on the side of the square. From 36 in. I can cut $\frac{36}{\frac{1}{2}\sqrt{3}}$ strips, $\frac{1}{2}\sqrt{3}$ in. wide, and $24\sqrt{3} = 41$ and a fraction;

∴ altogether there can be cut (36 + 35)41, or 2911 triangles. Ans.

Note.—The working, as given in the Hints of the earlier editions, is worked for triangles whose sides are 2 inches.

41. The area left is

$1296 - 2911 \times \frac{1}{4} \times \sqrt{3}$,

or $1296 - 1260{\cdot}5000 \ldots$

or 35·5 . . . sq. in. Ans.

42. As the cardboard is square, it matters not whether we cut the first row with their angles or sides on the top side.

Draw a figure, and let us have the sides on the top side, and the angle on the left side of the square. Between the angles of 2 hexagons we must leave 1 inch for the top sides of those in the second row; ∴ there can be only 12 hexagons in the first row, and 11 in the second, but they only occupy the breadth of a hexagon and a half, or $\frac{3\sqrt{3}}{2}$ in., and 3 rows occupy the breadth of 2 hexagons, and so on.

Now 36 contains $\frac{\sqrt{3}}{2}$ (see Example 40) 41 times; ∴ there can be 40 rows with 12 hexagons in 20 of the rows, and 11 hexagons in the other 20, or 240 + 220, or 460 hexagons. *Ans.*

43, 44. See Hints, p. 376.

45. $2^2 : 4^2 = £5 : £x$.

$$x = £\frac{5 \times 4 \times 4}{2 \times 2},$$

$$= £20.$$ Ans.

46. Call the base 1. Then the altitude of the triangle, which is one-sixth of the hexagon, is $\frac{\sqrt{3}}{2}$; and that of the triangle, which is one-eighth of the octagon, is $\frac{1}{2}\left(\frac{2}{\sqrt{2}} + 1\right)$, $\frac{1}{2}(\sqrt{2} + 1)$; ∴ ratio required is $\sqrt{3} : \sqrt{2} + 1$. Ans.

47. Draw a figure. Let A be the nail, and BC the rings. Draw AD perpendicular to BC.

Area surrounded is $\frac{1}{2} \times 12 \times 18$, or 108 sq. in. Ans.

48. See **47.**

$$AC = \sqrt{12^2 + 9^2}$$

$$= 15 \text{ ft.};$$

∴ string = 15 + 15 + 18,

or 48 in.

If this be lengthened by 20 in., there are 68 in., and the sides of the triangle we have to find the area of, are 25, 25, and 18, or the area

$$= \sqrt{34 \times 9 \times 9 \times 16}$$
$$= 3 \times 3 \times 4 \sqrt{34} \text{ sq. in.}$$
$$= 36 \sqrt{34} \text{ sq. in.}$$

$$\begin{array}{r|l} & 34{\cdot}0000 \,(5{\cdot}830 \\ & 25 \\ \hline 108 & 900 \\ & 864 \\ \hline 1163 & 3600 \\ & 3489 \\ \hline 1166 & 11100 \end{array}$$

Ans. 209·88 . . . sq. in.

49, 50. See Hints, p. 377.

CHAPTER XXV.

1, 2, 3. See Hints, p. 378.

4. Draw a regular hexagon in a circle, and you will see that a side is equal to the radius of the circle; and the perimeter of the circle is evidently greater than that of the hexagon, which is 6 times the radius, or 3 times the diameter, or π is more than 3. But if you draw a regular octagon whose sides are all equal to the radius of the circle, the octagon is entirely without the circle; $\therefore$ its perimeter or 4 times the diameter is greater than that of the circle, or π is less than 4.

5. *This problem, as it stood in earlier editions, is not possible.* Let ABC be one of the triangles into which the figure is divided, by joining the opposite points, bisect BC in D. Then the angle DAC is $\frac{1}{2}$ of $\frac{360}{20}$ or 9°.

And for an angle of 9° the ratio CD : CA is ·1564 . . and $\frac{5}{8}$ ft. × ·1564 is ·2606 ft., and not $\frac{1}{40}$ of 10 ft., or ·25. Correcting the perimeter to 10·424 ft., we obtain the area thus. First, to find AD.

$$AD = \sqrt{(\tfrac{5}{8})^2 A - (\cdot 2606)^2}$$
$$= \sqrt{2{\cdot}70986541}$$
$$= 1{\cdot}6762 \text{ . . ft.;}$$

$\therefore$ the entire area

$$= \tfrac{1}{2} \times 1{\cdot}6762 \times 10{\cdot}424,$$

or 8·7363544 . . sq. ft. Ans.

6, 7, 8. See Hints, p. 378.

9. Bisect the angle BAC by a straight line AD, which also bisects the chord BC.

Then, since we know the angle BAD to be 22·91°, we know from the tables AB : BD = 10000 : 3985 BD = 3·188, and the chord = 6·376 ft. Ans.

10. Circ. : 4 ft. = 360° : $x°$

$\pi \cdot 16$ ft. : 4 ft. = 360° : $x°$

$$x° = \frac{360}{4 \times \pi}$$

$$= 90 \times \cdot 31831$$

$$= 28 \cdot 6479°;$$

$\therefore$ the angle AOC = 28° 38′ .. and AOD = 14° 19′,

and for this angle

OA : AD = 10000 : 2502

8 : AD = 10000 : 2502

AD = 2·0016;

$\therefore$ OD = $\sqrt{64 - 4 \cdot 0164}$;

$\therefore$ OD = $\sqrt{59 \cdot 9836}$;

= 7·74 ft.;

$\therefore$ BD = 8 − 7·74,

or ·26 ft. Ans.

11. The area of circle

$= \pi \cdot 6^2$ sq. ft.;

$\therefore$ area of sector

$: \pi \cdot 6^2 = 3 : 2\pi \cdot 6$

$$\text{area} = \frac{\pi \cdot 6^2 3}{2 \cdot \pi \cdot 6} = 9 \text{ sq. ft.}$$ Ans.

12. $\pi \times 2 : 1 = 360° : x°$

$$x = \frac{360}{2 \times \pi}$$

$$= 180 \times \cdot 31831$$

$$= 57° \ 17' \ 47'' \text{ nearly.}$$

13. Area of circle

$= \pi \cdot 100^2$ sq. miles,

area of sector

$: \pi \cdot 100^2 = 45° : 360°$,

area of sector = $\frac{1}{8} \cdot \pi \cdot 10000$

= $\frac{1}{8}$ of 31415·9 sq. miles

= 3926·9875 sq. miles. Ans.

14. Area of circle = $\pi \cdot 10^2$

area of sector,

$= \pi \cdot 10^2 = 15\frac{1}{8} : 360$,

area of sector

$$= \frac{\pi \times 100 \times 46}{3 \times 360}, \text{ or } 129 \cdot 42685,$$

and the area of the triangular portion of sector is

$$\sqrt{34 \times 4 \times 4 \times 26},$$

or $4 \times 2 \sqrt{17 \times 13}$,

or $8 \times 14 \cdot 84$ sq. ft;

$\therefore$ area of segment

120·42685 − 118·72,

or 1·70685 sq. ft.

15. Area of sector

$: \pi \cdot 16 = 2 : \pi \times 2 \times 4$,

area of sector

$$= \frac{\pi \times 16 \times 2}{\pi \times 2 \times 4} = 4 \text{ sq. ft.}$$ Ans.

16. Diameter of larger circle

$$= \frac{100}{\pi} = 100 \times \cdot 31831$$

$$= 31 \cdot 831;$$

$\therefore$ diameter of smaller circle

$$= 25 \cdot 831;$$

$\therefore$ area of ring

$$\pi \times \tfrac{1}{4} \times (31 \cdot 831^2 - 25 \cdot 831^2)$$

$$= 3 \cdot 14159 \times \tfrac{1}{4} \times 57 \cdot 662 \times 6$$

$$= 271 \cdot 725 \text{ . . sq. ft.}$$

17. The smaller diameter

$= 4$ yds., or 12 ft.,

and that of the larger

4 yds. 4 ft., or 16 ft.,

area of ring

$= \pi(8^2 - 6^2)$ sq. ft.

$= \pi \times 14 \times 2$, or 28π sq. ft.

3·14159

28

2513272

628318

87·96452 sq. ft. Ans.

18. The diameter of the inside circle is $\frac{100}{\pi}$ ft.,

and the diameter of the other circles are

$(\frac{100}{\pi} + 4)$ ft., and $(\frac{100}{\pi} + 8)$ ft.,

area of the larger ring : area of *the smaller ring*

$$= \pi \left\{ \left(\frac{50}{\pi} + 4\right)^2 - \left(\frac{50}{\pi} + 2\right)^2 \right\}$$

$$: \pi \left\{ \left(\frac{50}{\pi} + 2\right)^2 - \left(\frac{50}{\pi}\right)^2 \right\}$$

$$= \left(\frac{100}{\pi} + 6\right)2 : \left(\frac{100}{\pi} + 2\right)2$$

$$= 100 + 6\pi : 100 + 2\pi$$

$= 118 \cdot 84954 : 106 \cdot 28318$,

which is the same answer as given in the Hints.

Note.—The student will see that to find the difference of the squares in either case, I multiplied the sum of the numbers by their difference.

19. The diameter of outside ring is $\frac{10}{\pi}$; $\therefore$ area of large ring is $\pi \left\{ \left(\frac{5}{\pi}\right)^2 - \left(\frac{5}{\pi} - 1\right)^2 \right\}$,

or $\pi \times \left(\frac{10}{\pi} - 1\right)1$;

and $\therefore$ the area of the smaller ring is $\frac{3}{10}$ of $\pi \times \left(\frac{10}{\pi} - 1\right)$,

or $3 - \frac{3\pi}{10}$, or $3 - \cdot 94277$,

or 2·05723 sq. ft. Ans.

20. Draw a straight line CODE. From C cut off CO

$= 8$, and $OQ = 6$. With centre O and distance OC describe a circle CAFB cutting CE in F; and with centre Q, distance QO, describe a circle OAEB, cutting the other circle in A and B, and COE in E. Join AB, cutting CE in D. The area of the triangle BOQ

$$= \sqrt{10 \times 2 \times 4 \times 4}$$
$$= 8\sqrt{5}$$
$$= 17{\cdot}888544 \text{ sq. ft};$$

but this triangle

$$= \tfrac{1}{2} \cdot OQ \times BD;$$
$$\therefore BD = 5{\cdot}962848 \text{ ft.}$$
$$\therefore AB = \underline{11{\cdot}925696 \text{ ft.}}$$

Ans.

21. The student will see that there are two lunes formed, one larger than the half of the larger circle, and one smaller than the half of the smaller. Let us find the area of the smaller. Drawing a figure as in **20**, we must first find the area of the sector OAFB. The table tells us that the angle AOB contains 96° 20′; $\therefore$ its area : $64 \times \pi = 96\frac{1}{3} : 360$, or 53·8028 sq. ft. The area of the triangle OAB is

$$\sqrt{13{\cdot}9628 \times (5{\cdot}9628)^2 \times 2{\cdot}0371},$$

or $5{\cdot}9628 \times 5{\cdot}333$, or 31·7996124 sq. ft.; $\therefore$ area of segment AFB = 22·1032 sq. ft.

To find segment AEB; area : $\pi \times 36 = 180\frac{2}{3} : 360$, or 56·7576 sq. ft., and the triangle QAB

$$= \sqrt{11{\cdot}9256 \times (5{\cdot}9628)^2 \times {\cdot}0371},$$

or 0·6651 sq. ft.; $\therefore$ segment AEB = 57·4227 sq. ft.; $\therefore$ lune AEBF = 57·4227 sq. ft. − 22·1032 sq. ft.,

or $\underline{35{\cdot}3195 \text{ sq. ft.}}$ Ans.

22. See Hints, p. 397.

23. Since $\pi r^2 = 113{\cdot}09724$;
$\therefore r^2 = 35{\cdot}99986$
$= 36$ very nearly;
$\therefore r = 6$.
And diameter $= \underline{12 \text{ ft.}}$ Ans.

24. The area of the circle is $4^2 \times 3{\cdot}14157$, and that of the hexagon is $6 \times 2 \times \frac{1}{2} \times \sqrt{3}$; $\therefore$ area of circle : area of hexagon
$= 4^2 \times 3{\cdot}14159 : 4^2 \times 2{\cdot}59807$
$= 3{\cdot}14159 : 2{\cdot}59807$. Ans.

25. The area of the circle is 36 sq. ft. × 3·14159, and that of the hexagon is 36 sq. ft. × 2·59807; $\therefore$ the remainder is 36 sq. ft. × ·54352,
or $\underline{19{\cdot}56672 \text{ sq. ft.}}$ Ans.

26. A figure will show at once that the ratio is 1 : 2.
Ans.

27. The four semicircles make up the whole square and the star; ∴ the star is $= 4 \cdot \frac{1}{2}\pi \cdot 9$ sq. ft. $- 36$ sq. ft., or 20·54862 sq. ft. Ans.

28. The area of the grass is $\pi \times 50^2$ sq. yds., or 7854 sq. yds., and $\frac{1}{2}$ an acre or 2420 sq. yds. added on gives us 10274 sq. yds.; and if this is $= \pi r^2$, ∴ $r^2 = 3270 \cdot 306666$, and $r = 57 \cdot 18$ yds; ∴ breadth of the path is 7·18 yds. Ans

29. If r be the radius of the circle, $2r + \frac{1}{4} \cdot 2\pi r = 100$ ft.; $3 \cdot 57079r = 100$ ft., and radius $= 28 \cdot 005$ ft. Ans.

30. Let us suppose the wheel revolved once in 10 sec., and that it went 1 mile an hour; ∴ the circumference of the wheel is $\frac{1760 \times 3}{60 \times 6}$; ∴ diameter of wheel
$= \frac{44}{5} \times \cdot 318309$ ft.,
or 4·6685 ft. Ans.

31. Since their rates are as 5 : 3, the faster runner must have run round five times while the slower has run three; ∴ the course is $\frac{10}{5}$, or 2 miles. And the diameter is
$2 \times 1760 \times \cdot 318309$ yds.,
or 1120·4476.. yds. Ans.

32. The sector is $\frac{1}{4}$ of the circle whose circumference is 40 ft.; area $= \frac{1}{8}$ of that of the circle whose circumference is 40 ft.

The radius $= \frac{20}{\pi}$, and the area $= \pi \times \frac{400}{\pi^2}$, and the sector $= \frac{1}{8}$ of $\frac{400}{\pi}$, or $50 \times \cdot 318309$ sq. ft., or 15·91545 sq. ft.
Ans.

33. See Hints, p. 380.

34. If we call the length 5 and the breadth 3, there are 15 sq. units; ∴ each sq. unit contains $\frac{3 \times 4840}{15}$ sq. yds., or 968 sq. yds.; ∴ since unit $= 31 \cdot 11$ yds.; ∴ the breadth is 155·55 yds., and the length 93·33 yds., and the perimeter $238 \cdot 88 \times 2$ yds., and to find this at 10s. a rod would cost $£\frac{24888 \times 2 \times 2}{11 \times 100 \times 2}$, or £45, 5s.
Ans.

35. Draw a figure; from O centre of circle draw OD per-

pendicular to BC. Then OD $= \frac{1}{2}$ rod or 5 ft., and BD or $\frac{1}{2}$ the side $= 5\sqrt{3}$ ft.

Area $= \frac{1}{2}$AD × BC
$= \frac{3}{2} \times 10 \times 5\sqrt{3}$
$= 129{\cdot}90381$ sq. ft.
Ans.

36. Let r = radius;

$$\therefore r = \frac{\sqrt{8.4.3.1}}{8},$$

and the area $= \pi r^2$,

or $\frac{8.4.3.1}{64}\pi$ or $\frac{3}{2}\pi$,

or $4{\cdot}71240$ sq. ft. Ans.

37. The rhombus is double an equilateral triangle whose sides are 4 ft.,

or $2 \times 2 \times 2 \times \sqrt{3}$ sq. ft.,
or $8 \times 1{\cdot}7320508$ sq. ft.,
or $13{\cdot}85640$ sq. ft. Ans.

38. Area of the triangle is $\sqrt{21.8.7.6}$, or 7.3.4, or 84 sq ft.; the area of the circle whose perimeter is 42 ft. is $21^2 \times {\cdot}318309$ sq. ft.; ∴ area of triangle : area of circle
$= 4 : 21 \times {\cdot}318309$
$= 4 : 6{\cdot}684489$. Ans.

39. The radius of the semicircle is $\frac{100}{5{\cdot}14159}$ or 19·46 ft., and its area
$= \frac{1}{2} \times \pi \times 19{\cdot}46 \times 19{\cdot}46$,
or 594 entire square ft. Ans.

40. $\frac{1760 \times 3}{300}$ is the number of ft. in the circumference; ∴ its diameter is $17{\cdot}6 \times {\cdot}318309$,
or $5{\cdot}602$ ft. Ans.

41. A figure will show that it is $\frac{1}{2}$ of 144 sq. ft.,
or 72 sq. ft. Ans.

42. The largest circle that can be inscribed is one whose diameter is 10 ft., and whose area is $\pi \times 25$ sq. ft.,
or $78{\cdot}039$ sq. ft. Ans.

43. If the student draw a figure he will see that the area of the square is 4 times that of the triangle whose sides are as $1 : 2 : \sqrt{5}$ when $\sqrt{5}$ represents 8 ft. If $\sqrt{5}$ represent 8 ft., 1 represents $\frac{8}{\sqrt{5}}$ ft., and the square required is 4 times the square on this unit, or $\frac{4 \times 64}{5}$ sq. ft., or $51\frac{1}{5}$ sq. ft. Ans.

44. Since 6 ft. will be the diagonal of the square, the area of the square is $\frac{1}{2} \times 36$,
or 18 sq. ft. Ans.

45. The area

$$= \tfrac{1}{2} \cdot s \times \frac{s}{2} \times \sqrt{3},$$

and if this = 100 sq. yds.,

$$s^2 = \frac{4}{\sqrt{3}} \times 100 \text{ sq. yds.}$$

$$= \frac{4 \times \sqrt{3} \times 100}{3} \text{ sq. yds.}$$

$$= 133\tfrac{1}{3} \times 1{\cdot}7320508$$

$$= 230{\cdot}9401;$$

$\therefore s = 15{\cdot}19$ yds.,

or 45·57 ft. Ans.

46. 5 sq. in. : 1 sq. in. = 100000 ac. : 20000 ac., and 20000 ac. $= \frac{20000}{640}$ sq. mls.; if $\therefore \frac{2000}{64}$ sq. mls. is represented by 1 sq. in., $\frac{10\sqrt{20}}{8}$ mls. is represented by 1 in., or the scale is 5·59 mls. to the in. Ans.

47. The square described about the square is double that inscribed in the square, and if this difference be 400 sq. yds., the area of the inscribed square is 400 sq. yds., and its side 20 yds. A figure will at once show that the radius must be $10\sqrt{2}$ yds.,

or 14·142136 yds. Ans.

48. The segment $= \frac{1}{6}$ of area of circle less that of the equilateral triangle whose sides are each 10 ft.,

or 52·3388 . . .

less 43·3012

9·0376

9·03 . . . sq. ft. Ans.

49, 50. See Hints, p. 380.

CHAPTER XXVI.

1. The prism contains 5×4 cub. ft.; $\therefore$ the pyramid contains $\frac{1}{3}$ of 5×4, or $6\frac{2}{3}$ cub. ft. Ans.

2. The student is advised to draw a figure.

Let AO be the perpendicular on to the base from the vertex, and let BCDE be the base of the pyramid;

$\therefore AB = \sqrt{AO^2 + OB^2}$.

But $OB^2 = \frac{1}{4} \times 2 \times 36$;

$\therefore AB = \sqrt{16 + 18}$

$= \sqrt{34}$

$= 5{\cdot}8309519$ ft. Ans.

3. Using the figure of **2**, we can find OB to be $\sqrt{100-36}$, or 8 ft.; ∴ the diagonal of the square is 16, and the area of the square is $\frac{1}{2} \times 16 \times 16$ sq. ft.; ∴ the solid content of the pyramid is $\frac{1}{3} \times 6 \times \frac{1}{2} \times 16 \times 16$, or 256 cub. ft. Ans.

4. The area of the base is $\frac{1}{2} \times 16 \times 16$ or 128 sq. ft., and the side of the base is $8\sqrt{2}$, and from A to the middle point of side of base is

$$\sqrt{100-32}, \text{ or } 2\sqrt{17} \text{ ft.}$$

∴ the area of side is

$$2\sqrt{17} \times 4\sqrt{2}, \text{ or } 8\sqrt{34} \text{ sq. ft.,}$$

or 46·6476152 sq. ft.;

∴ the four sides contain 186·5904608, to which if we add 128, we get the entire area, 314·5904608 sq. ft. Ans.

5. The length of the slant side of the cone is

$$\sqrt{36+4}, \text{ or } 2\sqrt{10} \text{ ft.;}$$

$$\therefore \text{ area} = \tfrac{1}{2} \times 2\sqrt{10} \times 4\pi$$
$$= 12{\cdot}64911108 \times \pi$$
$$= 39{\cdot}738446 \text{ sq. ft.}$$

Ans.

6. $\frac{1}{3}$ of $6 \times 4\pi$, or 8π, or 25·132 . . . cub. ft. Ans.

7. If radius = 8 ft., diameter = 16, and circumference = $16 \times \pi$; ∴ area of sphere = $16^2 \times \pi$, or 804·2496 . . . sq. ft. Ans.

8. The solid content is

$$\frac{16^3 \times \pi}{6} \text{ cub. ft.,}$$

or $\frac{1}{3}$ of $2048 \times \pi$ cub. ft. = 2144·6656 . . . cub. ft. Ans.

9. Area of base = $\pi \times 64$ sq. ft., and solid content of cylinder is $\pi \times 64 \times 16$ cub. ft., or 3216·9984 . . . cub. ft. Ans.

10. See Hints, p. 381.

11. If $\pi d^2 = 1$ acre,

$$d^2 = 4840 \times 9 \times 144 \times {\cdot}318309 \text{ sq. in.;}$$
$$\therefore\ d = 2 \times 11 \times 3 \times 12\sqrt{10 \times {\cdot}318309} \text{ in.}$$
$$= 792\sqrt{3{\cdot}18309} \text{ in.}$$
$$= 1413 \text{ in. Ans.}$$

12. $\frac{(1413)^3 \times \pi}{6}$ cub. in.,

or $\frac{9^3 \times 157^3 \times \pi}{6 \times 12^3 \times 3^3}$ cub. yds.,

or $\frac{157^3 \times \pi}{6 \times 4^3}$ cub. yds.,

or $\frac{3869893 \times \pi}{384}$,

or 31660·5621 . . . cub. yds. Ans.

13. The area of the base is $9 \times 2{\cdot}5981$ sq. ft.; ∴ the solid content is $9 \times 6 \times 2{\cdot}5981$ cub. ft., or 140·2974 . . . cub. ft. Ans.

14. The area of the base is $4\sqrt{3}$, and the altitude of the pyramid can be found thus—

Let O be the foot of perpendicular on base BCD; then OE, the perpendicular from O on to a side CD is $\frac{1}{3}$ of $2\sqrt{3}$.

And if AE = $\frac{5}{4}$ of AO,

since OE = $\frac{2}{3}\sqrt{3}$,

AO = $\frac{4}{3}$ of $\frac{2}{3}$ of $\sqrt{3}$;

∴ solid content is

$\frac{1}{3}$ of $4\sqrt{3}$ of $\frac{4}{3}$ of $\frac{2}{3}$ of $\sqrt{3}$ c. ft.,

or $3\frac{5}{9}$ cub. ft. Ans.

15. $\left(\frac{24000}{\pi}\right)$ is the diameter;

∴ solid content is

$\left(\frac{24000}{\pi}\right)^3 \times \frac{\pi}{6}$ cub. miles,

or $(24000)^3 \times \frac{1}{6} \times (\cdot 318309)^2$

$4224000000000 \times (\cdot 318309)^2$,

or 4279782966899·44 c. miles. Ans.

16. Cub. content is

$\frac{10^3 \times \pi}{6}$ cub. ft.,

and these weigh

$\frac{10^3 \times \pi \times 1000}{6 \times 16 \times 112}$ cwts.,

14 tons 12 cwts. 0 qr. 20 lbs. 14 oz. Ans.

17. The solid content is

$\frac{7^3}{4^3} \times \frac{\pi}{6}$,

and its weight is

$\frac{343}{64} \times \frac{\pi}{6} \times \frac{1820}{1000} \times \frac{1000}{1728}$ oz.,

or 31·179 . . . oz. Ans.

18. The cube weighs

$8 \times \frac{1820}{1000} \times \frac{1000}{1728}$ oz.,

and the sphere weighs

$8 \times \frac{\pi}{6} \times \frac{1820}{1000} \times \frac{1000}{1728}$;

∴ the part left weighs

$\left(1 - \frac{\pi}{6}\right) \times 8 \times \frac{1820}{1728}$ oz.,

or $\cdot 85841 \times 8 \times \frac{455}{432}$,

or 7·23289 oz. Ans.

19. The area of the triangle is $\sqrt{16 \times 6 \times 6 \times 4}$, or 48 sq. ft.; $\therefore$ solid content is 48×4, or 192 cub. ft. Ans.

20. The area of each of the four sides is $\sqrt{\frac{21}{2} \cdot \frac{5}{2} \cdot \frac{5}{2} \cdot \frac{11}{2}}$ sq. ft., or $\frac{5}{4}\sqrt{231}$ sq. ft., or $\frac{5}{4}$ of 15·1986 sq. ft., and 4 times this is 75·9930 sq. ft., or very nearly 76 ft. Ans.

21. If the area of the four sides is 76 sq. ft., that of each side is 19 sq. ft. We have to find two numbers which, multiplied together, are = 19, and whose squares, added together, are 64. If we add to 64 19×2, we have (as a figure would show) a square whose side is the sum of the two sides, or very nearly one of 10 ft.

If, again, we subtract from 64 19×2, we have the square on the difference of the two numbers, or one of very nearly 5 ft.; $\therefore$ one of the sides is $\frac{10+5}{2}$, or $7\frac{1}{2}$ ft., and the other $\frac{10-5}{2}$, or $2\frac{1}{2}$ ft.; the base, therefore, is a square, or 5 ft. Ans.

22. The perpendicular height of the prism is $8\sqrt{3}$; $\therefore$ its solid content is $6 \times 6 \times 8 \times \sqrt{3}$ cub. ft., or $288 \times 1{\cdot}73205$ cub. ft., or 498·73040 cub. ft. Ans.

23.

$$\frac{10 \times 1760 \times 3 \times 2 \times 9 \times 1000}{12 \times 12}$$

$$\text{or } \frac{5 \times 1760 \times 9 \times 1000}{12 \times 16 \times 112 \times 20} \text{ tons,}$$

or 184 tons 3 cwts. 0 qrs. 4 lbs. Ans.

24. The area of base is $\pi \times 16$. The cubic content of the water is $\pi \times 64 \times 2$ cub. ft. Its weight is

$$\frac{\pi \times 64 \times 2 \times 1000}{16 \times 112} \text{ cwts.}$$

= 11 tons 4 cwts. 1 qr. 1·68 lbs. Ans.

25. $\frac{29 \times 1 \times 13 \times 1000}{1728 \times 16}$ lbs.

$= 13\frac{2107}{3456}$ lbs. Ans.

26. Area of globe $= 5^2 \times \pi$ sq. in., and this × result of **25** will give us Ans.

27. $\frac{50 \times 12 \times 2 \times 1000}{1728 \times 16}$ lbs.,

or 4 tons 15 cwts. 2 qrs. 13 lbs. 6 oz. nearly. Ans.

28. The radius of the hemisphere is 1 ft.; $\therefore$ height of column is 7 ft.; and its solid content is $\pi \times 7$, and the solid content of hemisphere is

$\frac{1}{2} \times 2^3 \times \frac{\pi}{6}$, and of pillar $\pi(7 + \frac{2}{3})$,

or 24·0856 . . . cub. ft. Ans.

29. The length of the circumference of the plane is 10π; $\therefore$ the circumference of the base of cone is 2π, and its radius 1 ft. We have then a right-angled triangle, of which we know two sides to be 5 ft. and 1 ft.; $\therefore$ the other side is $\sqrt{25-1}$, or $2\sqrt{6}$ ft. Ans.

30. If its circumference is 2 ft., its radius is $\frac{1}{2}$ of $\frac{2}{\pi}$, or $\frac{1}{\pi}$; $\therefore$ the length of the column is $\left(10-\frac{2}{\pi}\right)$, and its cub. content is $\left(10-\frac{2}{\pi}\right)\times\frac{1}{\pi}$, and the two ends together make a sphere whose radius is $\frac{1}{\pi}$;

$\therefore$ its cubical content is

$$\left(\frac{2}{\pi}\right)^3\frac{\pi}{6}, \text{ or } \frac{4}{3\pi^2};$$

$\therefore$ the entire content is

$$\frac{10}{\pi}-\frac{2}{\pi^2}+\frac{4}{3\pi^2},$$

or 3·11613 . . . cub. ft. Ans.

31. Its area is

$$\left(10-\frac{2}{\pi}\right)2+\left(\frac{2}{\pi}\right)^2\pi \text{ sq. ft.},$$

or 20 sq. ft. Ans.

32. Circumference of cylinder $= 2.2.\pi$ in., and this is contained in 10 ft. $\frac{10\times12}{2\times2\times\pi}$, or 9·5493 . . . times. Ans.

33. If the solid content of a cube is 1 cub. ft., each side contains 1 sq. ft.; $\therefore$ its area is 6 sq. feet.

If the solid content of a sphere is 1 cub. ft.;

$$\therefore \frac{\text{diam.}^3\times\pi}{6}=1 \text{ cub. ft.},$$

$$\text{diam.}=\sqrt[3]{\frac{6}{\pi}},$$

and its area $=\text{diam.}^2\times\pi$;

$$\therefore \text{its area}=\left(\sqrt[3]{\frac{6}{\pi}}\right)^2\times\pi$$

$$=\sqrt[3]{\frac{36\times\pi^3}{\pi^2}}$$

$$=\sqrt[3]{36\times\pi}$$

$$=4\cdot83\ \ldots;$$

$\therefore$ area of cube : area of sphere $= 6 : 4\cdot83$. Ans.

34. 2·1816

27

152712

43632

58·9032 cub. in. Ans.

35. Since $s^3\times2\cdot1816=1$

$$s=\sqrt[3]{\frac{1}{2\cdot1816}} \text{ ft.}$$

$$=\sqrt[3]{\cdot458379}\ \ldots$$

$=$ ·7710 . . ft. Ans.

36. See Hints, p. 382.

37. The frustum contains $\frac{1}{3}$ of $\pi \times 16 \times 10 - \frac{1}{3}$ of $\pi \times \frac{64}{25} \times 4$,

or $\frac{\pi}{3}(160 - \frac{256}{25})$,

or $\pi \times \frac{3744}{75}$ cub. ft.,

or 148·8286 . . Ans.

38. See Hints, p. 383.

39. The solid content of the frustum is $\frac{1}{3}$ of

$$\left\{\left(\frac{6}{2\pi}\right)^2\pi \times 4 - \left(\frac{6}{8\pi}\right)^2\pi \times 1\right\},$$

or $\frac{1}{3}\left\{\frac{36}{\pi} - \frac{9}{16\pi}\right\}$,

or $(12 - \frac{3}{16}) \times \cdot 318309$,

or 3·76003 . . cub. ft. Ans.

40. See Hints, p. 383.

41. If the student draws a figure, he will see that the additional height is the hypotenuse of a triangle, one of whose sides is 1, and whose angle opposite this side is the semi-vertical angle of the cone. Hence this proportion will give us the side which is the extra height, viz.—

$$3 : \sqrt{10^2 + 3^2} = \tfrac{1}{12} : x \text{ ft.}$$

$$x = \frac{\sqrt{109} \times 12}{36} \text{ inches}$$

$$= 3.471 \ldots \text{ Ans.}$$

42. See Hints, p. 383.

43. The solid content of the entire pyramid would have been

$\frac{1}{3} \times 100^2 \times \sqrt{2} \times 100$ cub. yds.,

or $\frac{100^3 \times \sqrt{2}}{3}$,

and that of the pyramid left (since similar solids vary as the cubes of their homologous elements),

$$\frac{20^3}{3^3 \times (\sqrt{2})^3 \times 100^3} \text{ of } \frac{100^3 \times \sqrt{2}}{3} \text{ cub. yds., or } \frac{20^3}{3^4 \times 2};$$

$\therefore$ solid content of the frustum is $\frac{100^3\sqrt{2}}{3} - \frac{20^3}{3^4 \times 2}$ cub. yds.,

or $\frac{100^3 \times \sqrt{2} \times 3^3 - 20^2 \times 10}{3^4}$

cub. yds.,

or $\frac{38180767 \cdot 2}{81}$ cub. yds.,

or $\frac{381807672 \times 4}{810 \times 121}$ cub. poles.,

or 15582·39 . . cub. poles. Ans.

44. Let AO be drawn perpendicular to base of pyramid. Let E be middle point of one of the sides of the base. Join OE. From AO cut off AF = 20 ft., and draw FG to meet face parallel to OE. Through G draw HGK parallel to base,

First we must find AE, the altitude of the face. This will be found to be 150 yds.

The following proportion will give us the altitude of triangle AHK., viz.,

$\sqrt{2} \times 100 : 150 = \frac{20}{3}$ yds. $: x$ yds.;

and the following proportion will give us FG, which is half HK.

$\sqrt{2} \times 100 : 50 = \frac{20}{3}$ yds $: y$ yds.;

$\therefore$ the area of AHK is

$$\frac{150 \times 20 \times 50 \times 20}{3 \times \sqrt{2} \times 100 \times 3 \times \sqrt{2} \times 100},$$

or $\frac{50}{3}$ sq. yds.,

and the area of the face is 150×50, or 7500 sq. yds.;

$\therefore$ area of frustum is

$$4(7500 - \tfrac{50}{3}),$$

or $\frac{4 \times 22450}{3}$ sq. yds.,

and this $\times \frac{1}{36}$ gives us the cubic content of the layer of marble, viz.,

$$\frac{4 \times 22450}{3 \times 36} \text{ cub. yds.},$$

or $\underline{\frac{22450}{27} \text{ cub. yds.}}$ Ans.

45. In the cylinder there are $\pi \times 10$ cub. ft., which weigh $\frac{\pi \times 10 \times 20 \times 1000}{16}$ lbs., and will make

$$\frac{\pi \times 10 \times 20 \times 1000 \times 50}{16} \text{ coins},$$

or $\cdot 625000 \times \pi$ coins.

or $\underline{1963500 \text{ coins.}}$

46. The solid content of each sphere is $1^3 \times \frac{\pi}{6}$ cub. in., or $\frac{\pi}{6}$, and 1728 in. contains this $\frac{1728 \times 6}{\pi}$ times,

or $\underline{3300{\cdot}23808.}$ Ans.

47 to **49.** See Hints, p. 384.

50. The cubic content of the original cylinder is

$$\pi \times (\tfrac{1}{2})^2 \times 2 \text{ cub. in.}$$

To find out the thickness of this when rolled into one a mile long, we can ignore the outer coating altogether, as it cannot affect it.

If we call the new diameter d in.;

$$\therefore \pi.\left(\frac{d}{2}\right)^2 \times 1760 \times 3 \times 12 = \pi \times (\tfrac{1}{2})^2 \times 2;$$

$$\therefore d^2 = \frac{1 \times 2 \times 4}{4 \times 1760 \times 3 \times 12} = \frac{1}{12 \times 12 \times 220} = \frac{1}{31680} \text{ sq. in.};$$

$\therefore d = \underline{\cdot 000031 \ldots \text{ in.}}$ Ans.

CHAPTER XXVII.

1 to **4**. See Hints, pp. 384, 385.

5. If £688, 14s. 8d. is equivalent in value to
17511 fr. 42½ c.
$\frac{17511{\cdot}425}{688{\cdot}733}$ will give the number of francs each £1 is worth,
viz. 25 fr. 42½ c. Ans.

6. See paragraph **11** of this chapter.
The value of the dollar in London is $\frac{100}{108\frac{1}{2}}$, or $\frac{200}{217}$ of 4½s., or £$\frac{9}{40}$; ∴ the dollar is worth £$\frac{45}{217}$, and \$11315$\frac{315}{908}$
$= £\frac{1018381 \times 45}{90 \times 217}$
$= £\frac{46093}{2}$, or £2346, 10s. Ans.

7. We can do this by proportion, thus—
£10 : £1000 = 121$\frac{16}{60}$ fl. : x fl.
$x = \frac{7276 \times 1000}{60 \times 16}$
= 12126 fl. 40 kr.,
or more simply thus—

fl.	kr.
121	16 × 100
	10
1212	40
	10
12126	40

8. He first receives 100 × 25 fr., and secondly $\frac{97}{100}$ of 100 × 25 mks., or 2425 mks. Ans.

9. For £100 or 2000s. he has received 2425 mks., or gained 425s., or £21, 5s. Ans.

10. The American dollar in England is worth $\frac{100}{108}$ of 4s. 6d.; ∴ he must pay

$$£\frac{1468 \times 100 \times 9}{108 \times 2 \times 20},$$

and since the sov. contains 22 carat gold, the weight of bar gold will be equal to the weight of the sov., which is

$$\frac{1468 \times 100 \times 9 \times 40 \times 12}{108 \times 2 \times 20 \times 1869} \text{ oz.},$$

or 78·544 . . . oz. Ans.

34. See Hints, p. 386.

35. We must first find the volume of air which weighs 5 cwts., which is

$$\frac{5 \times 112 \times 16 \times 10^5}{10^3 \times 125} \text{ cub. ft.,}$$

or $112 \times 16 \times 4$ cub. ft.

But the volumes vary inversely as their sp. gr.

$$\cdot0001 : \cdot00125 = 112 \times 16 \times 4 \text{ cub. ft.} : x \text{ cub. ft.}$$

$$x = \frac{112 \times 16 \times 4 \times 125 \times 10^4}{10^5},$$

or 89600 cub. ft. Ans.

36 to 39. See Hints, p. 386.

40. Since the volumes of the immersed parts will vary as their heights, and the volumes vary inversely as their sp. gr., we have

$$79 : 1 = 4 \text{ in.} : h \text{ in.};$$

$$\therefore h = \frac{4 \times 100}{79}$$

$$= 5\cdot06 \ldots \text{ ft. Ans.}$$

41. This is simply changing the proportion thus,—

$$5\cdot06 : 4 = 1 : \text{sp. gr.}$$

$$\text{sp. gr.} = \frac{4}{5\cdot06}$$

$$= \cdot79. \text{ Ans.}$$

42. 2 cub. ft. of water weigh 2000 oz.

2 cub. ft. of cork weigh $\frac{2000 \times 24}{100}$ oz.;

$\therefore$ weight required will be

$$2000 - \frac{2000 \times 24}{100} \text{ oz.,}$$

$$\text{or } \frac{2000 \times 76}{100} \text{ oz.,}$$

or 1520 oz. Ans.

43. The one loses 2 lbs., and the other 1 lb.; $\therefore$ the water displaced by the one is 2 lbs., and by the other is 1 lb.; $\therefore$ their sp. gr. are as $\frac{5}{2} : \frac{4}{1}$, or 5 : 8. Ans.

44. $1\cdot009 - 1 = \cdot009$

$1\cdot027 - 1\cdot009 = \cdot018$;

$\therefore$ the volumes must be 2 : 1, or twice as much fresh water as sea.

45. Since $1 : \frac{1}{4} = 1 : \cdot25$, sp. gr. of the upper ball is $\cdot25$.

Calling the volume 1, and S_2 the sp. gr. of the lower ball, we have

$$V \times \cdot25 + 3V \times \frac{S}{2}$$

$$= \left(V + \frac{3V}{2}\right) \times 1;$$

$$\therefore S = (2\tfrac{1}{2} - \tfrac{1}{4}) \div \tfrac{3}{2} = \tfrac{3}{2},$$

or 6 times $\frac{1}{4}$. Ans.

46. The weight of equal volumes of the fluids are

$$6 - 3, \text{ and } 6 - 4;$$

$\therefore S_1 : S_2 = 3 : 2.$ Ans.

47. The specific gravities of the fluids are as 2 : 4, or 1 : 2; ∴ specific gravity of the mixture is $\frac{3}{2}$.

If then the body loses 2 oz. in a fluid whose sp. gr. is 1, it will lose 3 oz. in a fluid whose sp. gr. is $\frac{3}{2}$; ∴ it will weigh 13 oz. Ans.

48. The water displaced by the box, etc., weighs 8 oz. and its own weight; but if the water displaced by 1 third of the box weigh 8 oz., it all weighs 24 oz.; and ∴ the weight is 16 oz. Ans.

49. $\frac{54 \times 1000 \times 88}{1728 \times 10}$ oz.,

or 275 oz. Ans.

50. See Hints, p. 387.

Miscellaneous Answers on Part III.

I.

1. See Hints, p. 387.

2.

1	0	00	223648543	(607
	6	36	216	
	6	36		
	6	72		
	12			
	6			
1	1800	1080000		7648543
	7	12649		7648543
	1807	1092649		

Hence to form the cube of 607

1	0	00	216
	6	36	
	6	36	
	6	72	
	12		
	6		
1	1800	1080000	216
	7	12649	7648543
	1807	1092649	

∴ the cube of 607 is 223648543. Ans.

3.

1	0	00	27
	3	9	
	3	18	
	3		
	6		
	3		
1	90	2700	27
	2	184	5768
	92	2884	
	2	188	
	94		
	2		
1	960	307200	32768
	9	8721	2843289
	969	315921	

∴ cube of 329 is 35611289

Proof.

		35611289	(329
1	0	00	27
	3	9	
	3	9	
	3	18	
	6		
	3		
1	90	2700	8611
	2	184	5768
	92	2884	
	2	188	
	94		
	2		
1	960	307200	2843289
	9	8721	2843289
	969	315921	

4. We may imagine the first 3 parts all equal, and containing 1 unit each. There are ·5 units in all, and 375 ÷ 5, or 75, will give us the 2nd part; hence the 4 parts are 73, 75, 77, 150. Ans.

5. 7)100000

01428

13264 (the remrs.)

17325

46231 (the remrs.)

Hence the remainder is

$$5+6+6+0+4,$$

or some sevens; that is, the number will divide by 7 without remainder. Ans.

6. The circumference of the smaller wheel is

$$\frac{\pi \times 16}{12} \text{ ft., or } \frac{\pi \times 4}{3},$$

or 4·1888 ft.

The larger wheel revolves

$$\frac{1760 \times 3}{15},$$

and the smaller $\frac{1760 \times 3}{4 \cdot 1888}$, and the difference between these is 1260 − 352, or 908 times. Ans.

7. The difference between $\frac{1}{2}$ and $\frac{1}{3}$ is $\frac{1}{6}$, and between 1 more and 8 less is 9; $\therefore$ $\frac{1}{6}$ of the number is 9, or the number is 54. Ans.

8. If we take $\frac{1}{2}$ the height of the smaller box as our unit, the dimensions of the smaller box are $6 \times 3 \times 2$, and its entire area is $2 \times 6 \times 3 + 2\ (2 \times 6 + 2 \times 3)$, or $36 + 36$, or 72 square units. The dimensions of the larger box are $12 \times 6 \times 2$, and its entire area is $2 \times 12 \times 6 + 2\ (2 \times 12 + 2 \times 6)$, or $144 + 72$ square units. Hence, equating the value of the tin, we have $216 : 72 = 40s. : x s.$

$$x = \frac{72 \times 40}{216} s. = 13s.\ 4d.$$ Ans.

II.

9. $\frac{1}{2}$ of $\frac{3}{5}$ cost $\frac{100 \times 100}{102}$;

$\therefore$ whole ship cost $\frac{10}{3}$ of $\frac{100 \times 100}{102}$,

or £326, 15s. $11\frac{10}{17}d.$

10. $\frac{1}{2} - \frac{1}{5} = \frac{5-2}{10} = \frac{3}{10}$.

If $\frac{3}{10}$ of a number $= 1000$, the number $= 3333\frac{1}{3}$. Ans.

11. If 3 is represented by 7, 8 will be $\frac{8}{3}$ of 7, or $\frac{56}{3}$; $\therefore$ the three numbers are

$$4 : 7 : 18\tfrac{2}{3}.$$

$\frac{4}{29\frac{2}{3}}$ of $623 = 84$,

$\frac{7}{29\frac{2}{3}}$ of $623 = 147$,

$\frac{18\frac{2}{3}}{29\frac{2}{3}}$ of $623 = 392$,

Ans. 84, 147, 392.

12. $\frac{1}{2} : \frac{1}{3} : \frac{1}{4} : \frac{1}{6}$.

Multiplying each by $2 \times 3 \times 2$, we have $6 : 4 : 3 : 2$. Ans.

13.

1·03
3
3·09 ft.
12
·02 in.)37·08 in.
1854

Ans. 1854 times.

14.

1000000
1999
998001

998001 (999
81

189 | 1880
1701

1989 | 17901
17901

Ans. 999.

15.

7)3167

7)452 seven 3 ones

7)64 (seven)^{2}s 4 sevens

7)9 (seven)^{3}s 1 (seven)^2

1 (seven)^4 2 (seven)^{3}s.

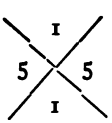
12143 (septenary)
12143 (septenary)

36462
51635
12143
24316
12143

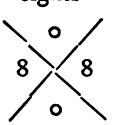
151152442 Ans.

16. To prove this we can either divide the product by 12143, when we ought to get a quotient of 12143 with no remainder, or we could find the square root, or we could cast out sixes and eights, and so test the answer. Let us do the last.

sixes



eights



Hence we know we are right or wrong by forty-eight.

III.

17. We must first find the amount.

£625
98·515625

723·515625

Now, divide by 625.

625)723·515625(1·157625
625

985
625

3601
3125

4765
4375

3906
3750

1562
1250

3125
3125

. . .

We now find the cube roo of this quotient.

1 0 00 1·157625(1·05
1 1 1

1 1
1 2

2
1

1	300	30000	157625
	5	1525	157625
	305	31525	

Ans. 5 per cent.

18. The discount is

$$\frac{100 \times £\frac{1}{20}}{4 \times \frac{1}{4}}, \text{ or } £5;$$

$\therefore$ the principal is $\frac{101}{1}$ of £5, or £505. Ans.

19, 20. See Hints, p. 388.

21. In the dearer paper each sq. inch cost $\frac{3 \times 12}{36 \times 23}$ d.; in the cheaper each sq. in. cost $\frac{3 \times 12}{36 \times 24}$ d., and the difference between these prices is $\frac{1}{23 \times 24}$ d.

As often as this fraction is contained in £1, so many sq. inches of wall are there, viz. $240 \times 23 \times 24$; and as often as this contains $(2 \times 16 + 2 \times 14)12$, or 60×12, so many inches are there in the height;

$$\therefore \text{ height} = \frac{240 \times 23 \times 25}{60 \times 12 \times 3} \text{ ft.}$$

$= \frac{46}{3}$, or 15 ft. 4 in. Ans.

22. $4 \times 8 \times 6$ cub. ft., or 192 cub. ft., and he could cut 48 slabs, 8 ft. $\times$ 6 ft., or 48×48, or 2304 sq. ft. Ans.

23. The principal is $\frac{105}{5}$ of £15, or £315. Ans.

24. See Hints, p. 388.

IV.

25. $\frac{2}{3}$ of £2 = £$\frac{4}{3}$,

$(\frac{1}{2} - \frac{1}{3})$ of $\frac{3}{5}$ of £$\frac{5}{2}$ = £$\frac{1}{6}$ of $\frac{3}{5}$ of $\frac{5}{2}$ = £$\frac{1}{4}$,

£$\frac{4}{3}$ − £$\frac{1}{4}$ = £$\frac{13}{12}$,

and £$\frac{13}{12}$ is $\frac{13}{60}$ of £5. Ans.

26, 27. See Hints, p. 388.

28. We must double the numerator and denominator of fraction that we may have 20 in the denominator, thus—

$$£3\frac{8\frac{1}{2}}{20}, \text{ or } £3, 8s. 4\tfrac{1}{2}d. \text{ Ans.}$$

29. Here we must multiply the numerator and denominator of whole fraction by 4, and of the resulting fraction $\frac{4}{6}$ by 2, thus—

$$£2\frac{1\frac{1}{6}}{5} = £2\frac{4\frac{8}{12}}{20}.$$

Ans. £2, 4s. 8d.

30.

1 mile : 1 ft. = 10 tons : x lbs.

$$x \text{ lbs.} = \frac{1 \times 10 \times 20 \times 112}{1760 \times 3}$$

$= 4\frac{8}{33}$ lbs. Ans.

31. 55 miles an hour

$$= \frac{55 \times 1760 \times 3 \times 4}{60 \times 60}$$

quarter feet a second, or $322\frac{2}{3}$. Ans.

32. Since 5 miles contains $7\frac{1}{2}$ ft.,

$$\frac{5 \times 1760 \times 3 \times 2}{15} \text{ times,}$$

or 3520 will be the number of linear units, and since 1 hr. contains 3 secs. 60×20 or 1200 times, this will give the number of time units; or the rate is 3520 units in 1200 units, or $2\frac{14}{15}$ units in 1 unit. Ans.

V.

33. See Hints, p. 388.

34. On £100 I pay $£94\frac{5}{8}$. I receive $£1\frac{1}{2} + £92\frac{7}{8}$; ∴ I lose $£\frac{1}{4}$ on £100 in 3 months. Ans.

35. The 5 per cents. cost me 108, and ∴ gives me an income of $\frac{5}{108}$ of my principal. But my former income was £30, and it is now only £25; I therefore have 5 cents., and they cost me £540, but this £540 I received for selling 10 cents.; ∴ their price is £54, but their real price is $£54\frac{1}{8}$, since I have to pay the broker the $\frac{1}{8}$ of his commission.

36. The entire area is 6 times the area of one of its sides, and the solid contents are the area multiplied by the number of ft. in the edge of the cube, but since the number of cubic ft. is double that of the square ft., the number must be 2×6, or 12.

The area of cube is 144×6 sq. ft., and the cubic contents are $144 \times 6 \times 2$, or 1728; ∴ Ans. is 12 ft.

37. $6^2 : (\frac{5}{3} \text{ of } 6)^2 = £40 : £x.$

$$x = £\frac{25 \times 36 \times 40}{9 \times 36},$$

or £111, 2s. $2\frac{2}{3}$d. Ans.

38. See Hints, p. 389.

5s. 2d. × 12, or 62 shillings, is the true value of what are nominally worth 66 shillings; ∴ a shilling is really worth $\frac{31}{33}$ of its nominal value. Ans.

39. This question must not be worked in connection with **38**, as 5s. 2d. is an exceptionally high price for silver.

See paragraph **6** of Chap. XXVII., and the answer is at once $\frac{1}{10}$ of 10000 fr., or 1000 fr., or £4. Ans.

If worked according to the data in **38**, the answer would be $\frac{2}{33}$ of 10000 or $606\frac{2}{33}$ francs.

40. See Hints, p. 389.

VI.

41. We must find the amount of $25{\cdot}32\frac{1}{2}$ fr. for 3 months at $4\frac{1}{2}$ per cent., or $25{\cdot}32\frac{1}{2} \times \frac{309}{800}$, or 25·96 nearly.

42. We must reduce $\frac{25{\cdot}17}{25{\cdot}27}$ to a fraction whose denominator is 1000,

$$\text{or } \tfrac{2517}{2527} = \frac{2517 \times 1000}{2527 \times 1000}$$

$$= 996{\cdot}1 \text{ nearly,}$$

$$\text{or } \underline{3\tfrac{9}{10} \text{ nearly.}} \text{ Ans.}$$

43. 1 cub. ft. of water : 1 cub. ft. of iron

$$= 1000 : 4\tfrac{1}{2} \times 1728,$$

$$= 1 : 7{\cdot}776;$$

$\therefore$ sp. gr. of iron $= \underline{7{\cdot}776}$. Ans.

44. $1728 : 1 \times 29\frac{1}{2} = 1000$ oz. $: x$ oz.

$$1 : 13{\cdot}6.$$

$$x = \frac{59 \times 136 \times 1000}{2 \times 10 \times 1728} \text{ oz.}$$

$$= \underline{232\tfrac{19}{108} \text{ oz.}} \text{ Ans.}$$

45. The area of the sphere whose diameter is 3 ft., or 36 in., is $36 \times \pi \times 36$ sq. in., and this $\times \frac{25075}{108}$ will give us the number of ounces which represents the pressure on the sphere. Calling π 3·1416, we have

$$\frac{36 \times 31416 \times 36 \times 25075}{108 \times 10000},$$

$$\text{or } \frac{7854 \times 3009}{25} \text{ oz.,}$$

$$\text{or } \underline{945307\tfrac{11}{25} \text{ oz.}} \text{ Ans.}$$

46. The area of the triangle is $9 \times {\cdot}43301$ sq. ft.; $\therefore$ its cubic contents are

$$\frac{389709}{6 \times 100000} \text{ cub. ft.}$$

Its weight is

$$\frac{389707}{6 \times 100000} \times \tfrac{78}{10} \times 1000 \text{ oz.,}$$

$$\text{or } \underline{2 \text{ cwts. } 3 \text{ qrs. } 8 \text{ lbs. } 10\tfrac{217}{1000} \text{ oz.}} \text{ Ans.}$$

47. This is really to divide 17 into two parts, so that one part $\times$ 9 and the other part $\times$ 10 are $= 17 \times 9{\cdot}25$.

$$17 \times 10 = 170,$$

$$\text{and } 17 \times 9{\cdot}25 = 157{\cdot}25,$$

$$\text{and the diff.} = 12{\cdot}75,$$

$$\text{and } 12{\cdot}75 \div (10 - 9) = 12\tfrac{3}{4};$$

$\therefore$ the volumes are $\underline{12\frac{3}{4} \text{ and } 4\frac{1}{4}.}$ Ans.

48. $1 \times 1 + 1 \times {\cdot}825 = 2S$ when S represents the sp. gr. of the mixture;

$$\therefore S = \underline{{\cdot}9125.} \text{ Ans.}$$

VII.

49. The price he paid for the entire quantity was $\frac{100}{120}$ of $£73\frac{1}{2}$; $\therefore$ each gallon of wine cost him $£\frac{147 \times 100}{2 \times 56 \times 120}$, or $\underline{£1, \text{ 1s. } 10\frac{1}{2}\text{d.}}$ Ans.

50, 51. See Hints, p. 389.

52. Each book cost him $\frac{70}{100}$ of $\frac{12}{13}$ of marked price, and he receives $\frac{3}{4}$ of marked price; $\therefore$ his gain is

$\frac{3}{4} - \frac{70}{100}$ of $\frac{12}{13}$ of marked price

$= \frac{975 - 840}{4 \cdot 25 \cdot 13}$ "

$= \frac{135}{4 \times 25 \times 13} = \frac{27}{20 \times 13}$ "

We have then this proportion

$$\tfrac{70}{100} \text{ of } \tfrac{12}{13} : 100 = \frac{27}{20 \times 13} : x.$$

$$x = \frac{100 \times 27 \times 100 \times 13}{20 \times 13 \times 70 \times 12}$$

$= \underline{16\frac{1}{4} \text{ per cent.}}$ Ans.

53. See Hints, p. 389.

He pays $\frac{12}{13}$ of $\frac{70}{100}$ of marked price, but for what he receives cash, he only receives $\frac{3}{4}$ of marked price; $\therefore$ the books for which he received £100 in cash were marked $\frac{4}{3}$ of £100, or £$133\frac{1}{3}$; $\therefore$ they cost him $£\frac{70 \times 12 \times 400}{100 \times 13 \times 3}$, or £86, 3s. $0\frac{12}{13}$d.

The £100 he receives at end of 1 month are $\frac{80}{100}$ of the marked price; $\therefore$ the marked price is $\frac{100}{80}$ of £100, and these books cost him

$$£\frac{70 \times 12 \times 100 \times 100}{100 \times 13 \times 80},$$

or £80, 15s. $4\frac{8}{13}$d.

Similarly, the other books cost him

$$£\frac{70 \times 12 \times 100 \times 100}{100 \times 13 \times 85},$$

and $£\frac{70 \times 12 \times 100 \times 100}{100 \times 13 \times 90}$,

and $£\frac{70 \times 12 \times 100 \times 100}{100 \times 13 \times 95}$,

or £76, os. $4\frac{76}{221}$d.,

£71, 15s. $10\frac{10}{13}$d.,

£68, os. $3\frac{219}{247}$d.,

and these, added together, are £382, 15s. $0\frac{2260}{4199}$d. Ans.

And if the student will apply the method given in the Hints, p. 389, he will obtain this result, viz. $£\frac{48215830}{125597}$, which is the same as above.

54. B would receive

$$£\frac{42 \times 200}{100 + 200 + 300 + 450},$$

or £8. Ans.

55.

$$\begin{array}{r|l} & 14\cdot40 \,(3\cdot794\ldots \\ & 9 \\ \hline 67 & 540 \\ & 469 \\ \hline 749 & 7100 \\ & 6741 \\ \hline & 35900 \end{array}$$

Ans. $3\cdot794\ldots$

56. See Hints, p. 389.

VIII.

57, 58. See Hints, p. 389.

59. $\frac{3}{5} = \frac{3}{3+2}$

$= \frac{3 \times \frac{70}{2}}{3 \times \frac{70}{2} + 70} = \frac{105}{175}$. Ans.

60. $\frac{3}{8}$ of 80 are 30, and $\frac{5}{8}$ of 80 are 50; ∴ the ratio required is 30 : 50. Ans.

61. $3 \times 5 = 15$,

$\frac{375}{15} = 25 = 5^2$;

∴ ratio required

$3 \times 5 : 5 \times 15$, or 15 : 25. Ans.

62.

3)16851	3)39729
5617	13243
4018	11234
1599	2009
1223	1599
369	410
369	

G. C. M. $= 3 \times 41$, or 123. Ans.

63. Since $60 - 50 = 10$, and $80 - 60 = 20$; ∴ they must be mixed in the ratio of 2 to 1, or 200 lbs. of water at 50° with 100 lbs. at 80°. Ans.

64. We may consider that there are in the water $13 \times 10 \times 15$ units of heat; ∴ the heat in the water would raise the 1 lb. of mercury 1950°. Ans.

IX.

65. The diameter of the larger circle is $30 \times \frac{1}{\pi}$ ft., and its area $= \left(\frac{15}{\pi}\right)^2 \pi$, or $15^2 \times \cdot 31831$.

The area of the smaller circle is $(\frac{24}{12})^2 \pi$;

∴ area of ring is the difference between these, or $71 \cdot 61975 - 12 \cdot 56636$, or 59·05339 sq. ft. Ans.

66. Since A has $\frac{130}{200}$ of the profits, and B $\frac{70}{200}$ of the profits, their times must have been as 13 : 7, or B's money was in 28 weeks. Ans.

67, 68. See Hints, p. 390.

69. The present value of £735 due 10 months hence at 6 per cent. is

$£\frac{735 \times 100}{105}$, or £700.

The present value of £203 due 3 months hence at 6 per cent. is

$£\frac{203 \times 200}{203}$, or £200.

The present value of £309 due 6 months hence at 6 per cent. is

$$£\frac{309 \times 100}{103}, \text{ or } £300.$$

Therefore the present worth of the remainder is £200. The question is, when will this be worth £223 at 6 per cent., which we can find thus—

$\frac{100 \times 23}{200 \times 6}$, or 1 yr. 11 mo. Ans.

70. $312 - 6 = 306$,
$306 - 6 = 300$,
$\frac{300}{6} = 50$;
∴ numbers are 6, 50, 256. Ans.

71. See Hints, p. 390.

72. $357\frac{2}{3}$ fl. $= \frac{1073}{3} \times 2{\cdot}16$ fr.

$$= \frac{1073 \times 216}{300} \text{ fr.}$$

$$= \frac{1073 \times 216}{300 \times 4} \text{ roubles}$$

= 193 roubles 14 copecks. Ans.

X.

73. The area is that of two equilateral triangles, whose sides are 2 in.,
or $2 \times 1 \times \sqrt{3}$ sq. in.,
or 3·4641016 sq. in. Ans.

74. See Hints, p. 390.

75. $37^2 = 1369$.
$1369^2 = 1874161$,
and $37^5 = 69343957$. Ans.

76.

1	0	00	000	0000	69343957	(37
	3	9	27	81	243	
	3	9	27	81		
	3	18	81	324		
	6	27	108			
	3	27	162			
	9	54				
	3	36				
	12					
	3					
1	150	9000	270000	4050000	45043957	
	7	1099	70693	2384851	45043957	
	157	10099	340693	6434851		

Ans. 37.

77. $\frac{100 \times 100}{100 \times 5}$, or 20 yrs.

Ans.

78. See Hints, p. 390.

79. 1 × distance required = 30, or 30 ft. Ans.

80.
5)625
5)125 0
5)25 0
5)5 0
1 0;

∴ 10000 is the number expressed in the quinary scale, and its square root is 100 (quinary), or 25 (denary). Ans.

XI.

81. 150 − 112 = 38
112 − 0 = 112;

∴ the water must be to the whisky as 38 : 112, or 19 : 56, or $\frac{19}{56}$ gal. of water with each gal. of spirits. Ans.

82. $\frac{\text{1 lb. 10 dwts.}}{\text{1 oz. 10 dwts.}} = \frac{250}{50} = 5.$

£	s.	d.
3	7	$8\frac{1}{2}$
		5

£16 18 $6\frac{1}{2}$ Ans.

83. $\frac{\text{1s.}}{\text{£1}} = \frac{1}{20}.$

£	s.	d.
16	18	$6\frac{1}{2}$
		20

£338 10 10 Ans.

84. Let us take as our unit $\frac{1}{5 \cdot 4 \cdot 3}$, what A can do in 5 days; ∴ A does 12 units each day, B does 15 units each day, and C does 20 units each day. There are 12 × 30 units, and 12 × 30 contains 12 + 15 + 20, or 47, $\frac{12 \times 30}{47}$ times,

or $7\frac{31}{47}$ days. Ans.

85. See Hints, p. 390.

86. Each so-called lb. that he sells only weighs $\frac{15}{16}$ of a lb. But for a lb. he receives $\frac{120}{100}$, or $\frac{6}{5}$ of 2s., or $\frac{12}{5}$s. If 1 lb. ought to be sold for $\frac{12}{5}$s., $\frac{15}{16}$ of a lb. ought to be sold for $\frac{15}{16}$ of $\frac{12}{5}$, or $\frac{9}{4}$s., or 2s. 3d. a lb. Ans.

87. Volume of the cylinder is $\frac{1}{4}$ of the volume of water of equal weight.